CS 301 - Lecture 21
Turing Machine Variations

Fall 2008

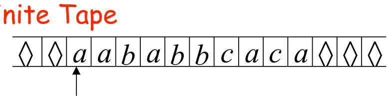
Review

- Languages and Grammars
  - Alphabets, strings, languages
- Regular Languages
  - Deterministic Finite and Nondeterministic Automata
  - Equivalence of NFA and DFA
  - Regular Expressions
  - Regular Grammars
  - Properties of Regular Languages
  - Languages that are not regular and the pumping lemma
- Context Free Languages
  - Context Free Grammars
  - Derivations: leftmost, rightmost and derivation trees
  - Parsing and ambiguity
  - Simplifications and Normal Forms
  - Nondeterministic Pushdown Automata
  - Pushdown Automata and Context Free Grammars
  - Deterministic Pushdown Automata
  - Pumping Lemma for context free grammars
  - Properties of Context Free Grammars
- Turing Machines
  - Definition, Accepting Languages, and Computing Functions
  - Combining Turing Machines and Turing's Thesis
  - Today: Turing Machine Variations

Variations of the Turing Machine

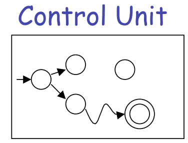
The Standard Model

Infinite Tape



Read-Write Head (Left or Right)

Control Unit



Deterministic

Variations of the Standard Model

Turing machines with:
- Stay-Option
- Semi-Infinite Tape
- Off-Line
- Multitape
- Multidimensional
- Nondeterministic

The variations form different Turing Machine **Classes**

We want to prove:

Each **Class** has the same power with the Standard Model

Same Power of two classes means:

Both classes of Turing machines accept the same languages

Same Power of two classes means:

For any machine M_1 of first class

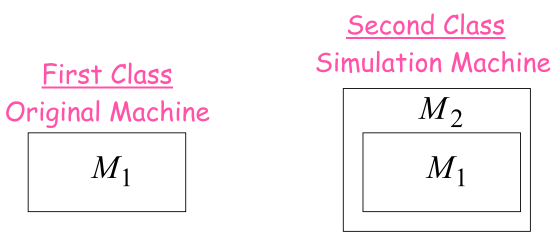
there is a machine M_2 of second class

such that: $L(M_1) = L(M_2)$

And vice-versa

Simulation: a technique to prove same power

Simulate the machine of one class
with a machine of the other class



Configurations in the Original Machine
correspond to configurations
in the Simulation Machine

Original Machine: $d_0 \succ d_1 \succ \cdots \succ d_n$

$\updownarrow \quad \updownarrow \quad \updownarrow$

Simulation Machine: $d'_0 \overset{*}{\succ} d'_1 \overset{*}{\succ} \cdots \overset{*}{\succ} d'_n$

Final Configuration

Original Machine: d_f

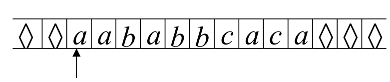
$\updownarrow$

Simulation Machine: d'_f

The Simulation Machine
and the Original Machine
accept the same language

Turing Machines with Stay-Option

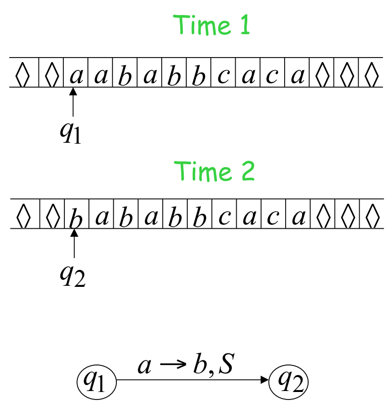
The head can stay in the same position



Left, Right, Stay

L,R,S: moves

Example:



Theorem: Stay-Option Machines have the same power with Standard Turing machines

Proof:

Part 1: Stay-Option Machines are at least as powerful as Standard machines

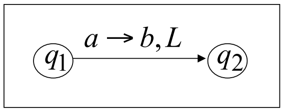
Proof: a Standard machine is also a Stay-Option machine (that never uses the S move)

Proof:

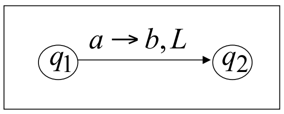
Part 2: Standard Machines are at least as powerful as Stay-Option machines

Proof: a standard machine can simulate a Stay-Option machine

Stay-Option Machine

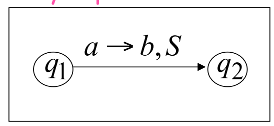


Simulation in Standard Machine

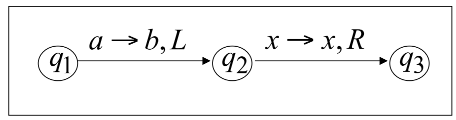


Similar for Right moves

Stay-Option Machine



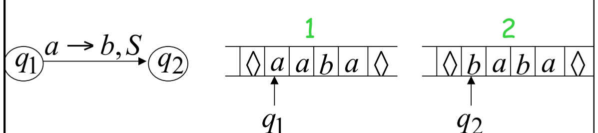
Simulation in Standard Machine



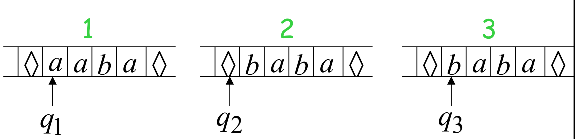
For every symbol x

Example

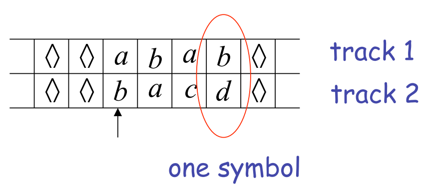
Stay-Option Machine:

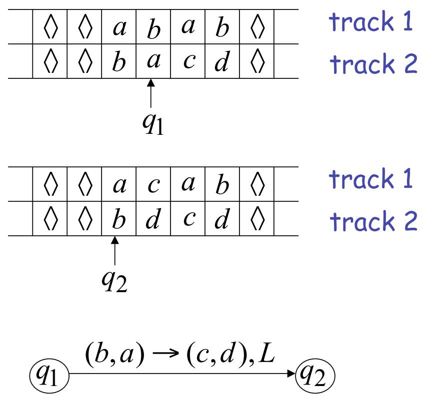


Simulation in Standard Machine:

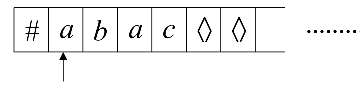


Standard Machine--Multiple Track Tape





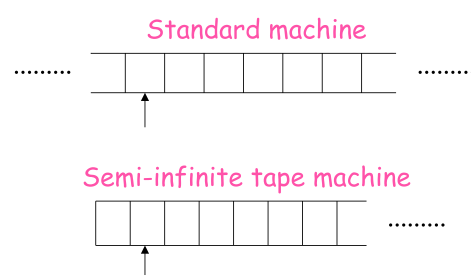
Semi-Infinite Tape

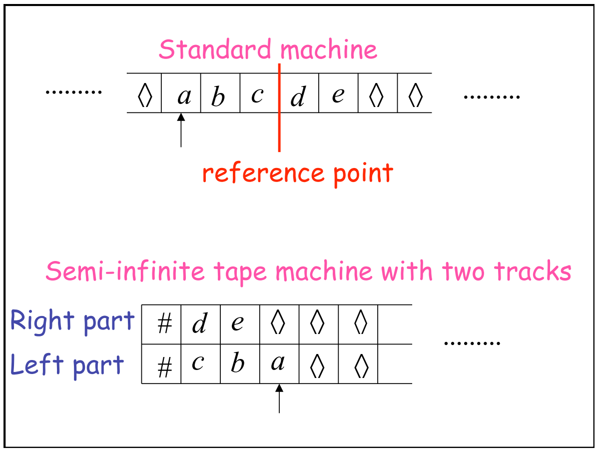


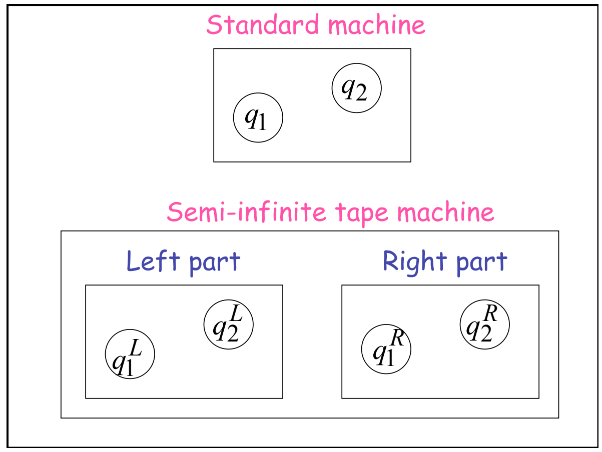
Standard Turing machines simulate
Semi-infinite tape machines:

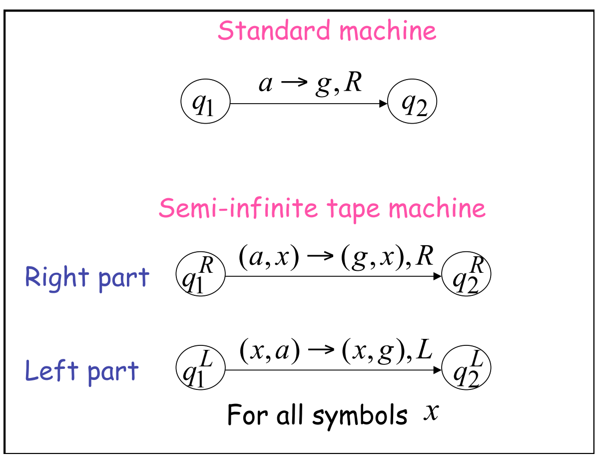
Trivial

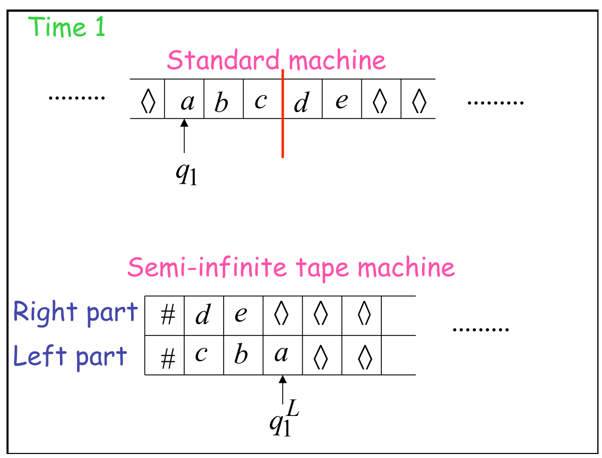
Semi-infinite tape machines simulate
Standard Turing machines:

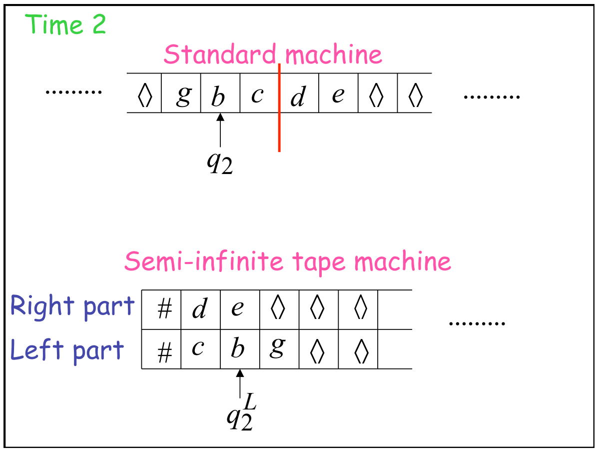


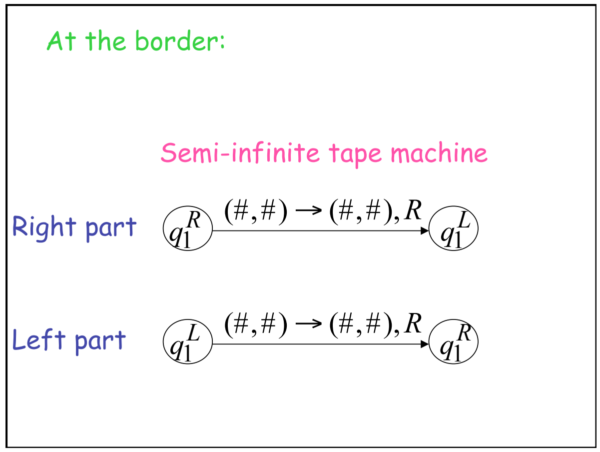

Standard machine
reference point
Semi-infinite tape machine with two tracks
Right part
Left part

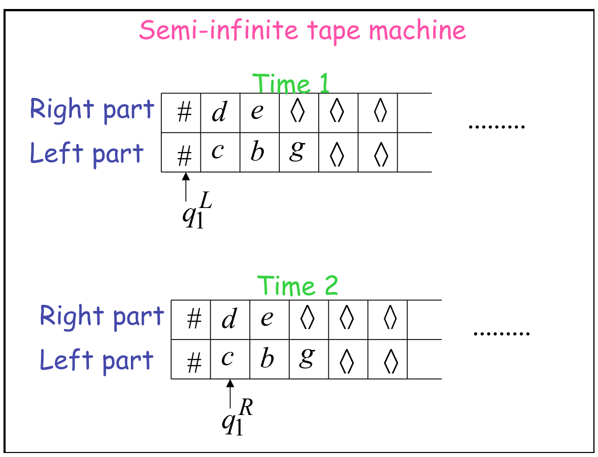

Standard machine
Semi-infinite tape machine
Left part
Right part


Standard machine
Semi-infinite tape machine
Right part
Left part
For all symbols x


Time 1
Standard machine
Semi-infinite tape machine
Right part
Left part

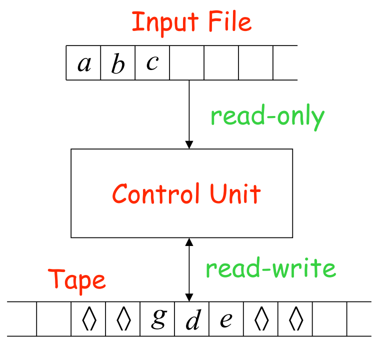






Theorem: Semi-infinite tape machines have the same power with Standard Turing machines

The Off-Line Machine



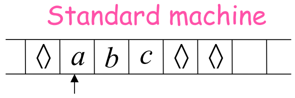
Off-line machines simulate Standard Turing Machines:

Off-line machine:

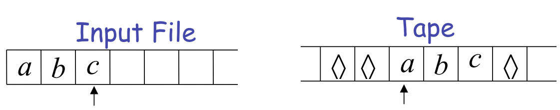
1. Copy input file to tape

2. Continue computation as in Standard Turing machine

Standard machine

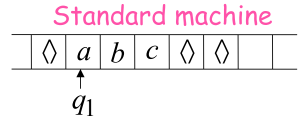


Off-line machine

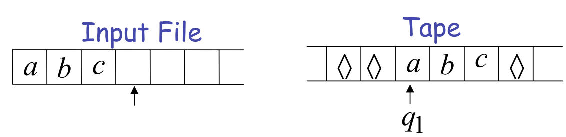


1. Copy input file to tape

Standard machine



Off-line machine

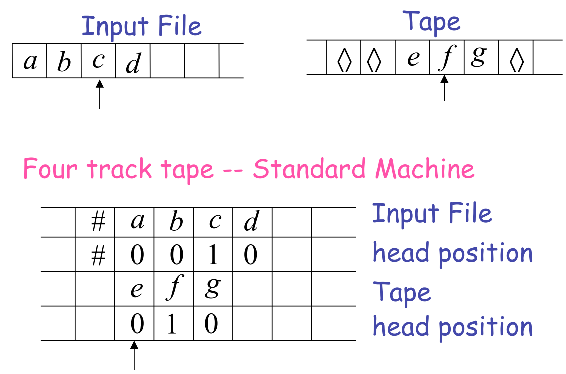


2. Do computations as in Turing machine

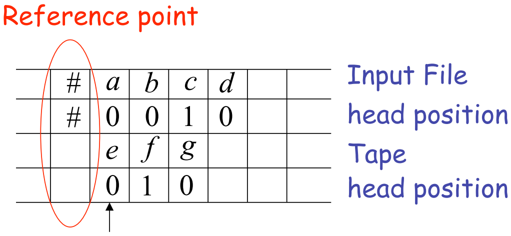
Standard Turing machines simulate
Off-line machines:

Use a Standard machine with four track tape
to keep track of
the Off-line input file and tape contents

Off-line Machine



Reference point

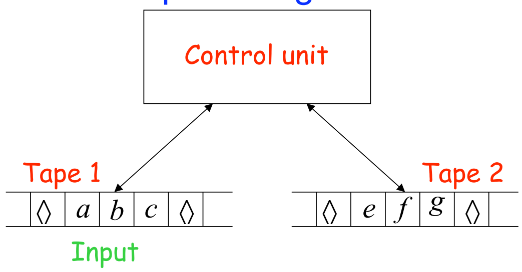


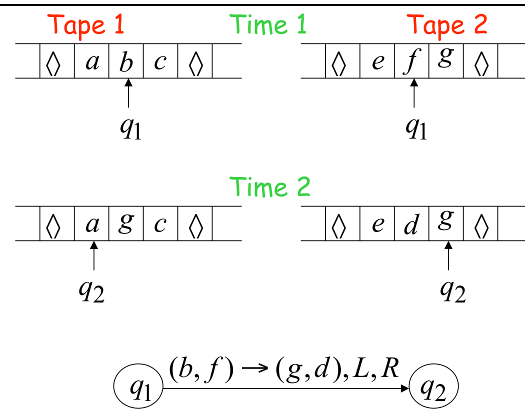
Repeat for each state transition:

- Return to reference point
- Find current input file symbol
- Find current tape symbol
- Make transition

Theorem: Off-line machines
have the same power with
Stansard machines

Multitape Turing Machines





Multitape machines simulate Standard Machines:

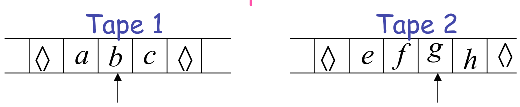
Use just one tape

Standard machines simulate Multitape machines:

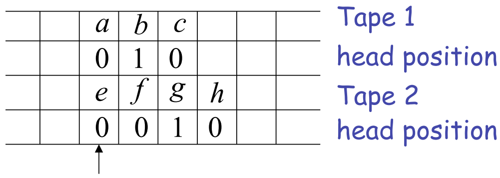
Standard machine:

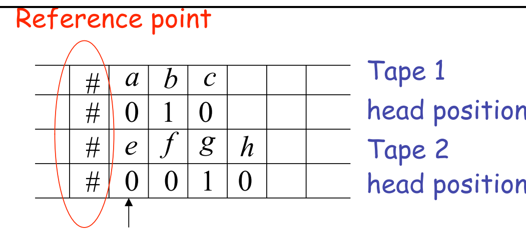
- Use a multi-track tape
- A tape of the Multiple tape machine corresponds to a pair of tracks

Multitape Machine



Standard machine with four track tape





Repeat for each state transition:

- Return to reference point
- Find current symbol in Tape 1
- Find current symbol in Tape 2
- Make transition

Theorem: Multi-tape machines have the same power with Standard Turing Machines

Same power doesn't imply same speed:

Language $L = \{a^n b^n\}$

	Acceptance Time
Standard machine	n^2
Two-tape machine	n

$L = \{a^n b^n\}$

Standard machine:

Go back and forth n^2 times

Two-tape machine:

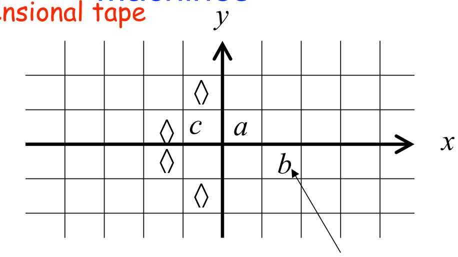
Copy b^n to tape 2 (n steps)

Leave a^n on tape 1 (n steps)

Compare tape 1 and tape 2 (n steps)

MultiDimensional Turing Machines

Two-dimensional tape



MOVES: L,R,U,D

U: up D: down

HEAD

Position: +2, -1

Multidimensional machines simulate Standard machines:

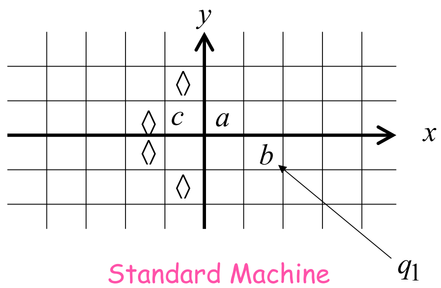
Use one dimension

Standard machines simulate Multidimensional machines:

Standard machine:

- Use a two track tape
- Store symbols in track 1
- Store coordinates in track 2

Two-dimensional machine



Standard Machine

a				b					c	
1	#	1	#	2	#	−	1	#	−	1

symbols

coordinates

↑ q_1

Standard machine:

Repeat for each transition

- Update current symbol
- Compute coordinates of next position
- Go to new position

Theorem: MultiDimensional Machines have the same power with Standard Turing Machines

What's Next

- Read
  - Linz Chapter 1,2.1, 2.2, 2.3, (skip 2.4), 3, 4, 5, 6.1, 6.2, (skip 6.3), 7.1, 7.2, 7.3, (skip 7.4), 8, 9, 10
  - JFLAP Chapter 1, 2.1, (skip 2.2), 3, 4, 5, 6, 7, (skip 8), 9
- Next Lecture Topics From 10.3, 10.4 and 10.5
  - Non-Deterministic Turing Machines, Universal Turing Machines, and Linear Bounded Automata
- Quiz 3 in Recitation on Wednesday 11/12
  - Covers Linz 7.1, 7.2, 7.3, (skip 7.4), 8, and JFLAP 5,6,7
  - Closed book, but you may bring one sheet of 8.5 x 11 inch paper with any notes you like.
  - Quiz will take the full hour
- Homework
  - Homework Thursday