CS440

Assignment 5 (due Thurs 11/15)

1. **[15 pts]** Let α, β be sentences in propositional logic. Prove each of the following assertions:

   (a) For any α, $False \vDash \alpha$.

   (b) α is valid (a tautology) if and only if $True \vDash \alpha$.

   In proving these statements use model checking, and recall that $\alpha \vDash \beta$ iff in every model in which α is true, β is also true (in other words, the set of models in which α is true is a subset of the set of models in which β is true).

2. **[20 pts]**
   For each of the following sentences in propositional logic, say whether it is satisfiable, unsatisfiable, or valid. Prove your answer.

   (a) $P \Rightarrow P$

   (b) $P \Rightarrow \neg P$

   (c) $\neg P \Rightarrow P$

   (d) $P \Leftrightarrow \neg P$

   (e) $P \Rightarrow (Q \Rightarrow P)$

3. **[10 pts]**
   Consider the following knowledgebase:
   $\neg A \cup \neg B$
   $A \Rightarrow B$.
   Use inference rules from propositional logic to show that this knowledgebase entails $\neg A$.

4. **[10 pts]**
   Convert the following propositional calculus sentences into CNF:

   (a) $\neg(((P \vee \neg Q) \Rightarrow R) \Rightarrow (P \wedge R))$

   (b) $P \Leftrightarrow (Q \wedge R)$

   You can use `logic.py` from the Russell and Norvig codebase to check your answer.

5. **[Resolution 25 pts]**
   You propose the following dice game to your friend: If I roll double 6, I win; with any other roll of the two dice, you lose. Express these statements (plus any other statements you might need) in propositional calculus. Use resolution to show that this

game is not very fair - you always win! Show all the steps in your proof. Use the `logic.py` from the Russell and Norvig codebase to test your knowledgebase, i.e. check that "I win" is entailed from the knowledge base, and provide in your writeup the code you used for that.

6. **First Order Logic [30 pts]**
In this question you will use first order logic to model relationships between US states. Use the following predicates and functions:

   - Borders(x, y): true if state x shares a common border with state y.
   - State(x): true if x is a US state.
   - Larger(x, y): true if x is a strictly larger state than state y or if city x is strictly larger than city y.
   - City(x): true if x is a city.
   - In(x, y): true if x is in state y.
   - Capital(x, y): true if x is the capital of state y.
   - Capital(x): a function that returns the capital of state x.

   Express the following English sentences in first order logic using the above predicates:

   (a) Texas borders three states.

   (b) The capital of Colorado is larger than the capital of Texas.

   (c) Alaska is the only US state that does not border any other state.

   (d) California is the largest state.

   (e) Every state has a capital.

   (f) Colorado has at least two cities in it.

Submission Submit a file named `a5.pdf` via Canvas. Due date: Sunday 11/15 at 11pm.