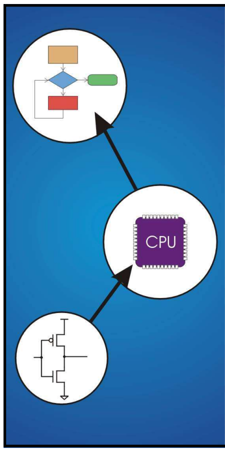

Chapter 2
Bits, Data Types, and Operations

How do we represent data in a computer?

At the lowest level, a computer is an electronic machine.

- **works by controlling the flow of electrons**

Easy to recognize two conditions:

1. **presence of a voltage – we'll call this state "1"**
2. **absence of a voltage – we'll call this state "0"**

Could base state on *value* of voltage, but control and detection circuits more complex.

- **compare turning on a light switch to measuring or regulating voltage**

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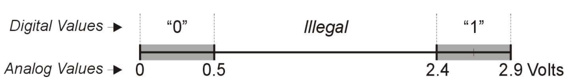
Computer is a binary digital system.

Digital system:
- finite number of symbols

Binary (base two) system:
- has two states: 0 and 1



Basic unit of information is the *binary digit*, or *bit*.

Values with more than two states require multiple bits.

- **A collection of two bits has four possible states: 00, 01, 10, 11**
- **A collection of three bits has eight possible states: 000, 001, 010, 011, 100, 101, 110, 111**
- ***A collection of n bits has 2^n possible states.***

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What kinds of data do we need to represent?

- **Numbers** – signed, unsigned, integers, floating point, complex, rational, irrational, …
- **Logical** – true, false
- **Text** – characters, strings, …
- **Instructions (binary)** – LC-3, x-86 ..
- **Images** – jpeg, gif, bmp, png ...
- **Sound** – mp3, wav..
- …

Data type:

- *representation* and *operations* within the computer

We'll start with numbers…

Unsigned Integers

Non-positional notation

- could represent a number ("5") with a string of ones ("11111")
- problems?

Weighted positional notation

- like decimal numbers: "329"
- "3" is worth 300, because of its position, while "9" is only worth 9

329

10^2 10^1 10^0

3x100 + 2x10 + 9x1 = 329

most significant 101 *least significant*

2^2 2^1 2^0

1x4 + 0x2 + 1x1 = 5

Unsigned Integers (cont.)

An *n*-bit unsigned integer represents 2^n values: from 0 to 2^n-1.

2^2	2^1	2^0	
0	0	0	0
0	0	1	1
0	1	0	2
0	1	1	3
1	0	0	4
1	0	1	5
1	1	0	6
1	1	1	7

Unsigned Binary Arithmetic

Base-2 addition – just like base-10!

- add from right to left, propagating carry

```
                   carry
  10010       10010        1111
+  1001     +  1011     +     1
  11011       11101       10000

              10111
            +   111
```

Subtraction, multiplication, division,…

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Signed Integers

With n bits, we have 2^n distinct values.

- assign about half to positive integers (1 through 2^{n-1}) and about half to negative (- 2^{n-1} through -1)
- that leaves two values: one for 0, and one extra

Positive integers

- just like unsigned – zero in ***most significant*** (MS) bit
  00101 = 5

Negative integers: formats

- sign-magnitude – set MS bit to show negative, other bits are the same as unsigned
  10101 = -5
- one's complement – flip every bit to represent negative
  11010 = -5
- in either case, MS bit indicates sign: 0=positive, 1=negative

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Two's Complement

Problems with sign-magnitude and 1's complement

- two representations of zero (+0 and –0)
- arithmetic circuits are complex
  - How to add two sign-magnitude numbers?
    - e.g., try 2 + (-3)
  - How to add to one's complement numbers?
    - e.g., try 4 + (-3)

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Two's Complement

Two's complement **representation developed to make circuits easy for arithmetic.**

- **for each positive number (X), assign value to its negative (-X), such that X + (-X) = 0 with "normal" addition, ignoring carry out**

```
  00101  (5)        01001  (9)
+ 11011  (-5)     + _____  (-9)
  00000  (0)        00000  (0)
```

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Two's Complement Representation

If number is positive or zero,

- **normal binary representation, zeroes in upper bit(s)**

If number is negative,

- **start with positive number**
- **flip every bit (i.e., take the one's complement)**
- **then add one**

```
  00101  (5)               01001  (9)
  11010  (1's comp)                (1's comp)
+     1                  +     1
  11011  (-5)                      (-9)
```

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Two's Complement Shortcut

To take the two's complement of a number:

- **copy bits from right to left until (and including) the first "1"**
- **flip remaining bits to the left**

```
  011010000                  011|010000
  100101111  (1's comp)  (flip)   (copy)
+         1
  100110000                  100|110000
```

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Two's Complement Signed Integers

MS bit is sign bit – it has weight -2^{n-1}.

Range of an n-bit number: -2^{n-1} through $2^{n-1} - 1$.

- The most negative number (-2^{n-1}) has no positive counterpart.

-2^3	2^2	2^1	2^0		-2^3	2^2	2^1	2^0	
0	0	0	0	0	1	0	0	0	-8
0	0	0	1	1	1	0	0	1	-7
0	0	1	0	2	1	0	1	0	-6
0	0	1	1	3	1	0	1	1	-5
0	1	0	0	4	1	1	0	0	-4
0	1	0	1	5	1	1	0	1	-3
0	1	1	0	6	1	1	1	0	-2
0	1	1	1	7	1	1	1	1	-1

Converting Binary (2's C) to Decimal

1. If leading bit is one, take two's complement to get a positive number.
2. Add powers of 2 that have "1" in the corresponding bit positions.
3. If original number was negative, add a minus sign.

$X = 01101000_{two}$
$= 2^6+2^5+2^3 = 64+32+8$
$= 104_{ten}$

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024

Assuming 8-bit 2's complement numbers.

More Examples

$X = 00100111_{two}$
$= 2^5+2^2+2^1+2^0 = 32+4+2+1$
$= 39_{ten}$

$X = 11100110_{two}$
$-X = 00011010$
$= 2^4+2^3+2^1 = 16+8+2$
$= 26_{ten}$
$X = -26_{ten}$

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024

Assuming 8-bit 2's complement numbers.

Converting Decimal to Binary (2's C)

First Method: *Division*

1. Find magnitude of decimal number. (Always positive.)
2. Divide by two – remainder is least significant bit.
3. Keep dividing by two until answer is zero, writing remainders from right to left.
4. Append a zero as the MS bit; if original number was negative, take two's complement.

$X = 104_{ten}$

104/2	=	52 r0	*bit 0*
52/2	=	26 r0	*bit 1*
26/2	=	13 r0	*bit 2*
13/2	=	6 r1	*bit 3*
6/2	=	3 r0	*bit 4*
3/2	=	1 r1	*bit 5*
1/2	=	0 r1	*bit 6*

$X = 01101000_{two}$

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Converting Decimal to Binary (2's C)

Second Method: *Subtract Powers of Two*

1. Find magnitude of decimal number.
2. Subtract largest power of two less than or equal to number.
3. Put a one in the corresponding bit position.
4. Keep subtracting until result is zero.
5. Append a zero as MS bit; if original was negative, take two's complement.

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024

$X = 104_{ten}$

104 - 64	=	40	*bit 6*
40 - 32	=	8	*bit 5*
8 - 8	=	0	*bit 3*

$X = 01101000_{two}$

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Operations: Arithmetic and Logical

Recall:
a data type includes *representation* and *operations*.

We now have a good representation for signed integers, so let's look at some arithmetic operations:

- Addition
- Subtraction
- Sign Extension

We'll also look at overflow conditions for addition.

Multiplication, division, etc., can be built from these basic operations.

Logical operations are also useful:

- AND
- OR
- NOT

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Addition

As we've discussed, 2's comp. addition is just binary addition.

- assume all integers have the same number of bits
- ignore carry out
- for now, assume that sum fits in n-bit 2's comp. representation

```
  01101000 (104)    11110110 (-10)
+ 11110000 (-16)  + ________ (-9)
  01011000 (98)              (-19)
```

Assuming 8-bit 2's complement numbers.

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Subtraction

Negate subtrahend (2nd no.) and add.

- assume all integers have the same number of bits
- ignore carry out
- for now, assume that difference fits in n-bit 2's comp. representation

```
  01101000 (104)    11110110 (-10)
- 00010000 (16)   - ________ (-9)
  01101000 (104)    11110110 (-10)
+ 11110000 (-16)  + ________ (9)
  01011000 (88)              (-1)
```

Assuming 8-bit 2's complement numbers.

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Sign Extension

To add two numbers, we must represent them with the same number of bits.

If we just pad with zeroes on the left:

4-bit	8-bit
0100 (4)	00000100 (still 4)
1100 (-4)	00001100 (12, not -4)

Instead, replicate the MS bit -- the sign bit:

4-bit	8-bit
0100 (4)	00000100 (still 4)
1100 (-4)	11111100 (still -4)

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Overflow

If operands are too big, then sum cannot be represented as an *n*-bit 2's comp number.

```
  01000 (8)       11000 (-8)
+ 01001 (9)     + 10111 (-9)
  10001 (-15)     01111 (+15)
```

We have overflow if:

- signs of both operands are the same, and
- sign of sum is different.

Another test -- easy for hardware:

- carry into MS bit does not equal carry out

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Logical Operations

Operations on logical TRUE or FALSE

- two states -- takes one bit to represent: TRUE=1, FALSE=0

A	B	A AND B
0	0	0
0	1	0
1	0	0
1	1	1

A	B	A OR B
0	0	0
0	1	1
1	0	1
1	1	1

A	NOT A
0	1
1	0

View *n*-bit number as a collection of *n* logical values

- operation applied to each bit independently

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Examples of Logical Operations

AND

- useful for clearing bits
  - AND with zero = 0
  - AND with one = no change

```
     11000101
AND  00001111
     00000101
```

OR

- useful for setting bits
  - OR with zero = no change
  - OR with one = 1

```
     11000101
OR   00001111
     11001111
```

NOT

- unary operation -- one argument
- flips every bit

```
NOT  11000101
     00111010
```

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Hexadecimal Notation

It is often convenient to write binary (base-2) numbers as hexadecimal (base-16) numbers instead.

- **fewer digits -- four bits per hex digit**
- **less error prone -- easy to corrupt long string of 1's and 0's**

Binary	Hex	Decimal	Binary	Hex	Decimal
0000	0	0	1000	8	8
0001	1	1	1001	9	9
0010	2	2	1010	A	10
0011	3	3	1011	B	11
0100	4	4	1100	C	12
0101	5	5	1101	D	13
0110	6	6	1110	E	14
0111	7	7	1111	F	15

Converting from Binary to Hexadecimal

Every four bits is a hex digit.

- **start grouping from right-hand side**

```
011 1010 1000 1111 0100 1101 0111
 ↓    ↓    ↓    ↓    ↓    ↓    ↓
 3    A    8    F    4    D    7
```

This is not a new machine representation, just a convenient way to write the number.

Fractions: Fixed-Point

How can we represent fractions?

- **Use a "binary point" to separate positive from negative powers of two -- just like "decimal point."**
- **2's comp addition and subtraction still work.**
  - **if binary points are aligned**

$2^{-1} = 0.5$
$2^{-2} = 0.25$
$2^{-3} = 0.125$

```
  00101000.101 (40.625)
+ 11111110.110 (-1.25)
  00100111.011 (39.375)
```

No new operations -- same as integer arithmetic.

Very Large and Very Small: Floating-Point

Large values: 6.023×10^{23} -- requires 79 bits
Small values: 6.626×10^{-34} -- requires >110 bits

Use equivalent of "scientific notation": $F \times 2^E$
Need to represent F (*fraction*), E (*exponent*), and sign.
IEEE 754 Floating-Point Standard (32-bits):

1b	8b	23b
S	Exponent	Fraction

$$N = (-1)^S \times 1.\text{fraction} \times 2^{\text{exponent}-127}, \quad 1 \le \text{exponent} \le 254$$
$$N = (-1)^S \times 0.\text{fraction} \times 2^{-126}, \quad \text{exponent} = 0$$

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Floating Point Example

Single-precision IEEE floating point number:

`1 01111110 10000000000000000000000`

sign exponent fraction

- **Sign is 1 – number is negative.**
- **Exponent field is 01111110 = 126 (decimal).**
- **Fraction is 0.100000000000… = 0.5 (decimal).**

Value = $-1.5 \times 2^{(126-127)} = -1.5 \times 2^{-1} = -0.75$.

2-29

Decimal to float32

1. **Change decimal number to binary**
2. **Move radix point so there is only a single 1 bit to the left of the radix point.**
   - **Every position moved to the left increases the exponent size by one.**
   - **Every position moved to the right decreases the exponent size by one.**
   - **The initial exponent is 0.**
3. **Remove leading 1 from resulting binary number and store this number in bits 0-22.**
4. **Add 127 to exponent and store binary representation of exponent in bits 23-30**
5. **Store sign in bit 31, 1 for negative, 0 for positive.**

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Float 32 to decimal

1. Check bit MSB (31) for sign, 1 negative, 0 positive
2. Extract bits 30 – 23, and find their value in binary then subtract 127 to get the exponent
3. Extract bits 22 – 0 and add implicit bit with value 1 to location 23 to get the fractional part
4. Change value of exponent to 0 by shifting radix point of fractional part right to reduce exponent and left to increase exponent
5. Convert resulting binary number to decimal

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Floating-Point Operations

Will regular 2's complement arithmetic work for Floating Point numbers?

(*Hint*: In decimal, how do we compute $3.07 \times 10^{12} + 9.11 \times 10^{8}$? Need to work with exponents)

2-32

Text: ASCII Characters

ASCII: Maps 128 characters to 7-bit code.

- both printable and non-printable (ESC, DEL, …) characters

00	nul	10	dle	20	sp	30	0	40	@	50	P	60	`	70	p
01	soh	11	dc1	21	!	31	1	41	A	51	Q	61	a	71	q
02	stx	12	dc2	22	"	32	2	42	B	52	R	62	b	72	r
03	etx	13	dc3	23	#	33	3	43	C	53	S	63	c	73	s
04	eot	14	dc4	24	$	34	4	44	D	54	T	64	d	74	t
05	enq	15	nak	25	%	35	5	45	E	55	U	65	e	75	u
06	ack	16	syn	26	&	36	6	46	F	56	V	66	f	76	v
07	bel	17	etb	27	'	37	7	47	G	57	W	67	g	77	w
08	bs	18	can	28	(	38	8	48	H	58	X	68	h	78	x
09	ht	19	em	29	)	39	9	49	I	59	Y	69	i	79	y
0a	nl	1a	sub	2a	*	3a	:	4a	J	5a	Z	6a	j	7a	z
0b	vt	1b	esc	2b	+	3b	;	4b	K	5b	[	6b	k	7b	{
0c	np	1c	fs	2c	,	3c	<	4c	L	5c	\	6c	l	7c	|
0d	cr	1d	gs	2d	-	3d	=	4d	M	5d	]	6d	m	7d	}
0e	so	1e	rs	2e	.	3e	>	4e	N	5e	^	6e	n	7e	~
0f	si	1f	us	2f	/	3f	?	4f	O	5f	_	6f	o	7f	del

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Interesting Properties of ASCII Code

What is relationship between a decimal digit ('0', '1', …) and its ASCII code?

What is the difference between an upper-case letter ('A', 'B', …) and its lower-case equivalent ('a', 'b', …)?

Given two ASCII characters, how do we tell which comes first in alphabetical order?

Unicode: 128 characters are not enough. 1990s Unicode was standardized, Java used Unicode.

No new operations -- integer arithmetic and logic.

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Other Data Types

Text strings
- sequence of characters, terminated with NULL (0)
- typically, no hardware support

Image
- array of pixels
  - monochrome: one bit (1/0 = black/white)
  - color: red, green, blue (RGB) components (e.g., 8 bits each)
  - other properties: transparency
- hardware support:
  - typically none, in general-purpose processors
  - MMX -- multiple 8-bit operations on 32-bit word

Sound
- sequence of fixed-point numbers

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LC-3 Data Types

Some data types are supported directly by the instruction set architecture.

For LC-3, there is only one hardware-supported data type:
- 16-bit 2's complement signed integer
- Operations: ADD, AND, NOT

Other data types are supported by <u>interpreting</u> 16-bit values as logical, text, fixed-point, etc., in the software that we write.

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