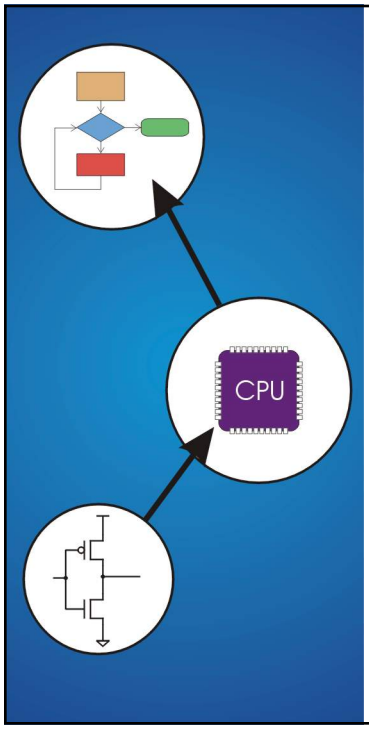



# Chapter 2
## Bits, Data Types, and Operations

### How do we represent data in a computer?

**At the lowest level, a computer is an electronic machine.**

- **works by controlling the flow of electrons**

**Easy to recognize two conditions:**

1. **presence of a voltage – we'll call this state "1"**
2. **absence of a voltage – we'll call this state "0"**

**Could base state on *value* of voltage,**
**but control and detection circuits more complex.**

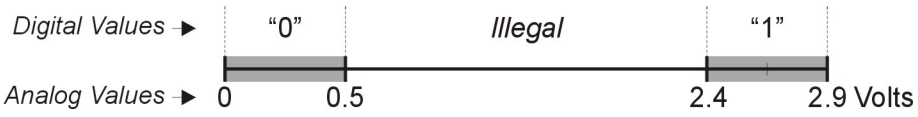
- **compare turning on a light switch to measuring or regulating voltage**

2-2

## Computer is a binary digital system.

Digital system:
- finite number of symbols

Binary (base two) system:
- has two states: 0 and 1




**Basic unit of information is the *binary digit*, or *bit*.**

**Values with more than two states require multiple bits.**

- **A collection of two bits has four possible states: 00, 01, 10, 11**
- **A collection of three bits has eight possible states: 000, 001, 010, 011, 100, 101, 110, 111**
- ***A collection of n bits has $2^n$ possible states.***

2-3

## What kinds of data do we need to represent?

- **Numbers – signed, unsigned, integers, floating point, complex, rational, irrational, …**
- **Logical – true, false**
- **Text – characters, strings, …**
- **Instructions (binary) – LC-3, x-86 ..**
- **Images – jpeg, gif, bmp, png ...**
- **Sound – mp3, wav..**
- **…**

**Data type:**
- ***representation* and *operations* within the computer**

**We'll start with numbers…**

2-4

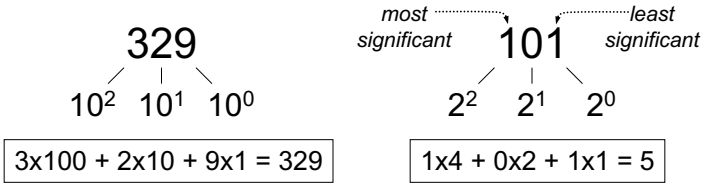
## Unsigned Integers

**Non-positional notation**

- could represent a number ("5") with a string of ones ("11111")
- problems?

**Weighted positional notation**

- like decimal numbers: "329"
- "3" is worth 300, because of its position, while "9" is only worth 9






## Unsigned Integers (cont.)

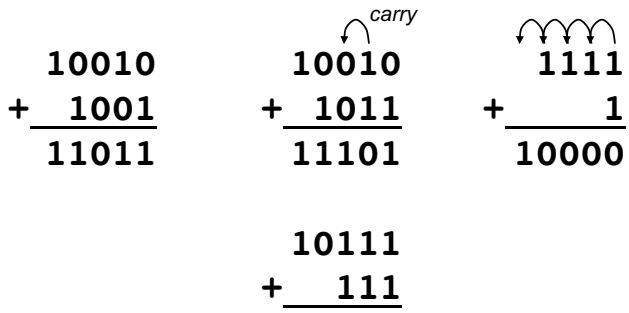
**An *n*-bit unsigned integer represents $2^n$ values: from 0 to $2^n$-1.**

| $2^2$ | $2^1$ | $2^0$ | |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 2 |
| 0 | 1 | 1 | 3 |
| 1 | 0 | 0 | 4 |
| 1 | 0 | 1 | 5 |
| 1 | 1 | 0 | 6 |
| 1 | 1 | 1 | 7 |

## Unsigned Binary Arithmetic

**Base-2 addition – just like base-10!**

- **add from right to left, propagating carry**




Subtraction, multiplication, division,…

2-7

## Signed Integers

**With n bits, we have $2^n$ distinct values.**

- **assign about half to positive integers (1 through $2^{n-1}$) and about half to negative (- $2^{n-1}$ through -1)**
- **that leaves two values: one for 0, and one extra**

**Positive integers**

- **just like unsigned – zero in *most significant* (MS) bit**
  00101 = 5

**Negative integers: formats**

- **sign-magnitude – set MS bit to show negative, other bits are the same as unsigned**
  10101 = -5
- **one's complement – flip every bit to represent negative**
  11010 = -5
- **in either case, MS bit indicates sign: 0=positive, 1=negative**

2-8

## Two's Complement

**Problems with sign-magnitude and 1's complement**

- two representations of zero (+0 and –0)
- arithmetic circuits are complex
  - ➢ How to add two sign-magnitude numbers?
    - e.g., try 2 + (-3)
  - ➢ How to add to one's complement numbers?
    - e.g., try 4 + (-3)

2-9

## Two's Complement

*Two's complement* **representation developed to make circuits easy for arithmetic.**

- for each positive number (X), assign value to its negative (-X), such that X + (-X) = 0 with "normal" addition, ignoring carry out

```
  00101  (5)        01001  (9)
+ 11011  (-5)     +       (-9)
  00000  (0)        00000  (0)
```

2-10

## Two's Complement Representation

**If number is positive or zero,**

- normal binary representation, zeroes in upper bit(s)

**If number is negative,**

- start with positive number
- flip every bit (i.e., take the one's complement)
- then add one

```
 00101  (5)          01001  (9)
 11010  (1's comp)          (1's comp)
+    1              +    1
 11011  (-5)               (-9)
```

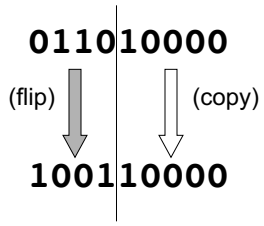
2-11

## Two's Complement Shortcut

**To take the two's complement of a number:**

- copy bits from right to left until (and including) the first "1"
- flip remaining bits to the left

```
 011010000
 100101111  (1's comp)
+        1
 100110000
```




2-12

## Two's Complement Signed Integers

**MS bit is sign bit – it has weight $-2^{n-1}$.**

**Range of an n-bit number: $-2^{n-1}$ through $2^{n-1} - 1$.**

- **The most negative number ($-2^{n-1}$) has no positive counterpart.**

| $-2^3$ | $2^2$ | $2^1$ | $2^0$ | |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 0 | 2 |
| 0 | 0 | 1 | 1 | 3 |
| 0 | 1 | 0 | 0 | 4 |
| 0 | 1 | 0 | 1 | 5 |
| 0 | 1 | 1 | 0 | 6 |
| 0 | 1 | 1 | 1 | 7 |

| $-2^3$ | $2^2$ | $2^1$ | $2^0$ | |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | −8 |
| 1 | 0 | 0 | 1 | −7 |
| 1 | 0 | 1 | 0 | −6 |
| 1 | 0 | 1 | 1 | −5 |
| 1 | 1 | 0 | 0 | −4 |
| 1 | 1 | 0 | 1 | −3 |
| 1 | 1 | 1 | 0 | −2 |
| 1 | 1 | 1 | 1 | −1 |

2-13

## Converting Binary (2's C) to Decimal

1. **If leading bit is one, take two's complement to get a positive number.**
2. **Add powers of 2 that have "1" in the corresponding bit positions.**
3. **If original number was negative, add a minus sign.**

$$X = 01101000_{two}$$
$$= 2^6+2^5+2^3 = 64+32+8$$
$$= 104_{ten}$$

| $n$ | $2^n$ |
|---|---|
| 0 | 1 |
| 1 | 2 |
| 2 | 4 |
| 3 | 8 |
| 4 | 16 |
| 5 | 32 |
| 6 | 64 |
| 7 | 128 |
| 8 | 256 |
| 9 | 512 |
| 10 | 1024 |

*Assuming 8-bit 2's complement numbers.*

2-14

## More Examples

$X = 00100111_{two}$
$= 2^5+2^2+2^1+2^0 = 32+4+2+1$
$= 39_{ten}$

$X = 11100110_{two}$
$-X = 00011010$
$= 2^4+2^3+2^1 = 16+8+2$
$= 26_{ten}$
$X = -26_{ten}$

| $n$ | $2^n$ |
|---|---|
| 0 | 1 |
| 1 | 2 |
| 2 | 4 |
| 3 | 8 |
| 4 | 16 |
| 5 | 32 |
| 6 | 64 |
| 7 | 128 |
| 8 | 256 |
| 9 | 512 |
| 10 | 1024 |

*Assuming 8-bit 2's complement numbers.*

2-15

## Converting Decimal to Binary (2's C)

**First Method:** *Division*

1. **Find magnitude of decimal number. (Always positive.)**
2. **Divide by two – remainder is least significant bit.**
3. **Keep dividing by two until answer is zero, writing remainders from right to left.**
4. **Append a zero as the MS bit; if original number was negative, take two's complement.**

$X = 104_{ten}$

| | | | |
|---|---|---|---|
| 104/2 | = | 52 r0 | *bit 0* |
| 52/2 | = | 26 r0 | *bit 1* |
| 26/2 | = | 13 r0 | *bit 2* |
| 13/2 | = | 6 r1 | *bit 3* |
| 6/2 | = | 3 r0 | *bit 4* |
| 3/2 | = | 1 r1 | *bit 5* |
| 1/2 | = | 0 r1 | *bit 6* |

$X = 01101000_{two}$

2-16

## Converting Decimal to Binary (2's C)

| *n* | $2^n$ |
|---|---|
| 0 | 1 |
| 1 | 2 |
| 2 | 4 |
| 3 | 8 |
| 4 | 16 |
| 5 | 32 |
| 6 | 64 |
| 7 | 128 |
| 8 | 256 |
| 9 | 512 |
| 10 | 1024 |

**Second Method:** *Subtract Powers of Two*

1. **Find magnitude of decimal number.**
2. **Subtract largest power of two less than or equal to number.**
3. **Put a one in the corresponding bit position.**
4. **Keep subtracting until result is zero.**
5. **Append a zero as MS bit; if original was negative, take two's complement.**

$X = 104_{ten}$

| | | | |
|---|---|---|---|
| 104 - 64 | = | 40 | *bit 6* |
| 40 - 32 | = | 8 | *bit 5* |
| 8 - 8 | = | 0 | *bit 3* |

$X = 01101000_{two}$

2-17

## Operations: Arithmetic and Logical

**Recall:**
**a data type includes *representation* and *operations*.**

**We now have a good representation for signed integers, so let's look at some arithmetic operations:**

- **Addition**
- **Subtraction**
- **Sign Extension**

**We'll also look at overflow conditions for addition.**

**Multiplication, division, etc., can be built from these basic operations.**

**Logical operations are also useful:**

- **AND**
- **OR**
- **NOT**

2-18

## Addition

**As we've discussed, 2's comp. addition is just binary addition.**

- **assume all integers have the same number of bits**
- **ignore carry out**
- **for now, assume that sum fits in n-bit 2's comp. representation**

```
  01101000  (104)     11110110  (-10)
+ 11110000  (-16)  +            (-9)
  01011000  (98)                (-19)
```

*Assuming 8-bit 2's complement numbers.*

2-19

## Subtraction

**Negate subtrahend (2nd no.) and add.**

- **assume all integers have the same number of bits**
- **ignore carry out**
- **for now, assume that difference fits in n-bit 2's comp. representation**

```
  01101000  (104)     11110110  (-10)
- 00010000  (16)   -            (-9)
  01101000  (104)     11110110  (-10)
+ 11110000  (-16)  +            (9)
  01011000  (88)                (-1)
```

*Assuming 8-bit 2's complement numbers.*

2-20

## Sign Extension

**To add two numbers, we must represent them with the same number of bits.**

**If we just pad with zeroes on the left:**

| 4-bit | | 8-bit | |
|---|---|---|---|
| 0100 | (4) | 00000100 | (still 4) |
| 1100 | (-4) | 00001100 | (12, not -4) |

**Instead, replicate the MS bit -- the sign bit:**

| 4-bit | | 8-bit | |
|---|---|---|---|
| 0100 | (4) | 00000100 | (still 4) |
| 1100 | (-4) | 11111100 | (still -4) |

2-21

## Overflow

**If operands are too big, then sum cannot be represented as an *n*-bit 2's comp number.**

| | | | |
|---|---|---|---|
| 01000 | (8) | 11000 | (-8) |
| + 01001 | (9) | + 10111 | (-9) |
| 10001 | (-15) | 01111 | (+15) |

**We have overflow if:**

- **signs of both operands are the same, and**
- **sign of sum is different.**

**Another test -- easy for hardware:**

- **carry into MS bit does not equal carry out**

2-22

## Logical Operations

**Operations on logical TRUE or FALSE**

- two states -- takes one bit to represent: TRUE=1, FALSE=0

| A | B | A AND B |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

| A | B | A OR B |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

| A | NOT A |
|---|---|
| 0 | 1 |
| 1 | 0 |

**View *n*-bit number as a collection of *n* logical values**

- operation applied to each bit independently

2-23

## Examples of Logical Operations

**AND**

- useful for clearing bits
  - AND with zero = 0
  - AND with one = no change

```
  11000101
AND 00001111
  --------
  00000101
```

**OR**

- useful for setting bits
  - OR with zero = no change
  - OR with one = 1

```
  11000101
OR  00001111
  --------
  11001111
```

**NOT**

- unary operation -- one argument
- flips every bit

```
NOT 11000101
  --------
  00111010
```

2-24

## Hexadecimal Notation

**It is often convenient to write binary (base-2) numbers as hexadecimal (base-16) numbers instead.**

- **fewer digits -- four bits per hex digit**
- **less error prone -- easy to corrupt long string of 1's and 0's**

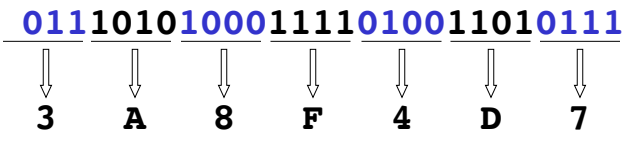
| Binary | Hex | Decimal | Binary | Hex | Decimal |
|---|---|---|---|---|---|
| 0000 | 0 | 0 | 1000 | 8 | 8 |
| 0001 | 1 | 1 | 1001 | 9 | 9 |
| 0010 | 2 | 2 | 1010 | A | 10 |
| 0011 | 3 | 3 | 1011 | B | 11 |
| 0100 | 4 | 4 | 1100 | C | 12 |
| 0101 | 5 | 5 | 1101 | D | 13 |
| 0110 | 6 | 6 | 1110 | E | 14 |
| 0111 | 7 | 7 | 1111 | F | 15 |

2-25

## Converting from Binary to Hexadecimal

**Every four bits is a hex digit.**

- **start grouping from right-hand side**



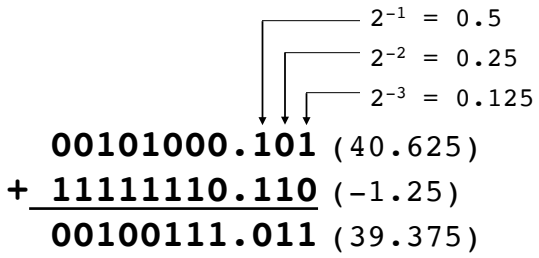

*This is not a new machine representation, just a convenient way to write the number.*

2-26

## Fractions: Fixed-Point

**How can we represent fractions?**

- **Use a "binary point" to separate positive from negative powers of two -- just like "decimal point."**
- **2's comp addition and subtraction still work.**
  - **if binary points are aligned**




*No new operations -- same as integer arithmetic.*

2-27

## Very Large and Very Small: Floating-Point

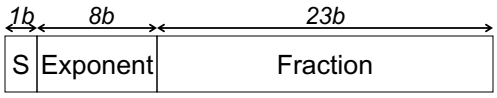
**Large values: $6.023 \times 10^{23}$ -- requires 79 bits**

**Small values: $6.626 \times 10^{-34}$ -- requires >110 bits**

**Use equivalent of "scientific notation": $F \times 2^E$**

**Need to represent F (*fraction*), E (*exponent*), and sign.**

**IEEE 754 Floating-Point Standard (32-bits):**



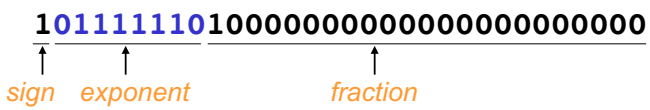

$$N = (-1)^S \times 1.\text{fraction} \times 2^{\text{exponent}-127},\ 1 \le \text{exponent} \le 254$$

$$N = (-1)^S \times 0.\text{fraction} \times 2^{-126},\ \text{exponent} = 0$$

2-28

## Floating Point Example

**Single-precision IEEE floating point number:**




- **Sign is 1 – number is negative.**
- **Exponent field is 01111110 = 126 (decimal).**
- **Fraction is 0.100000000000… = 0.5 (decimal).**

**Value = $-1.5 \times 2^{(126-127)} = -1.5 \times 2^{-1} = -0.75$.**

2-29

## Floating-Point Operations

**Will regular 2's complement arithmetic work for Floating Point numbers?**

**(*Hint*: In decimal, how do we compute $3.07 \times 10^{12} + 9.11 \times 10^{8}$?**
**Need to work with exponents )**

2-30

## Text: ASCII Characters

**ASCII: Maps 128 characters to 7-bit code.**

- **both printable and non-printable (ESC, DEL, …) characters**

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 00 | nul | 10 | dle | 20 | sp | 30 | 0 | 40 | @ | 50 | P | 60 | ` | 70 | p |
| 01 | soh | 11 | dc1 | 21 | ! | 31 | 1 | 41 | A | 51 | Q | 61 | a | 71 | q |
| 02 | stx | 12 | dc2 | 22 | " | 32 | 2 | 42 | B | 52 | R | 62 | b | 72 | r |
| 03 | etx | 13 | dc3 | 23 | # | 33 | 3 | 43 | C | 53 | S | 63 | c | 73 | s |
| 04 | eot | 14 | dc4 | 24 | $ | 34 | 4 | 44 | D | 54 | T | 64 | d | 74 | t |
| 05 | enq | 15 | nak | 25 | % | 35 | 5 | 45 | E | 55 | U | 65 | e | 75 | u |
| 06 | ack | 16 | syn | 26 | & | 36 | 6 | 46 | F | 56 | V | 66 | f | 76 | v |
| 07 | bel | 17 | etb | 27 | ' | 37 | 7 | 47 | G | 57 | W | 67 | g | 77 | w |
| 08 | bs | 18 | can | 28 | ( | 38 | 8 | 48 | H | 58 | X | 68 | h | 78 | x |
| 09 | ht | 19 | em | 29 | ) | 39 | 9 | 49 | I | 59 | Y | 69 | i | 79 | y |
| 0a | nl | 1a | sub | 2a | * | 3a | : | 4a | J | 5a | Z | 6a | j | 7a | z |
| 0b | vt | 1b | esc | 2b | + | 3b | ; | 4b | K | 5b | [ | 6b | k | 7b | { |
| 0c | np | 1c | fs | 2c | , | 3c | < | 4c | L | 5c | \ | 6c | l | 7c | \| |
| 0d | cr | 1d | gs | 2d | - | 3d | = | 4d | M | 5d | ] | 6d | m | 7d | } |
| 0e | so | 1e | rs | 2e | . | 3e | > | 4e | N | 5e | ^ | 6e | n | 7e | ~ |
| 0f | si | 1f | us | 2f | / | 3f | ? | 4f | O | 5f | _ | 6f | o | 7f | del |

2-31

## Interesting Properties of ASCII Code

**What is relationship between a decimal digit ('0', '1', …) and its ASCII code?**

**What is the difference between an upper-case letter ('A', 'B', …) and its lower-case equivalent ('a', 'b', …)?**

**Given two ASCII characters, how do we tell which comes first in alphabetical order?**

**Unicode: 128 characters are not enough. 1990s Unicode was standardized, Java used Unicode.**

*No new operations -- integer arithmetic and logic.*

2-32

## Other Data Types

**Text strings**

- sequence of characters, terminated with NULL (0)
- typically, no hardware support

**Image**

- array of pixels
  - monochrome: one bit (1/0 = black/white)
  - color: red, green, blue (RGB) components (e.g., 8 bits each)
  - other properties: transparency
- hardware support:
  - typically none, in general-purpose processors
  - MMX -- multiple 8-bit operations on 32-bit word

**Sound**

- sequence of fixed-point numbers

2-33

## LC-3 Data Types

**Some data types are supported directly by the instruction set architecture.**

**For LC-3, there is only one hardware-supported data type:**

- 16-bit 2's complement signed integer
- Operations: ADD, AND, NOT

**Other data types are supported by interpreting 16-bit values as logical, text, fixed-point, etc., in the software that we write.**

2-34