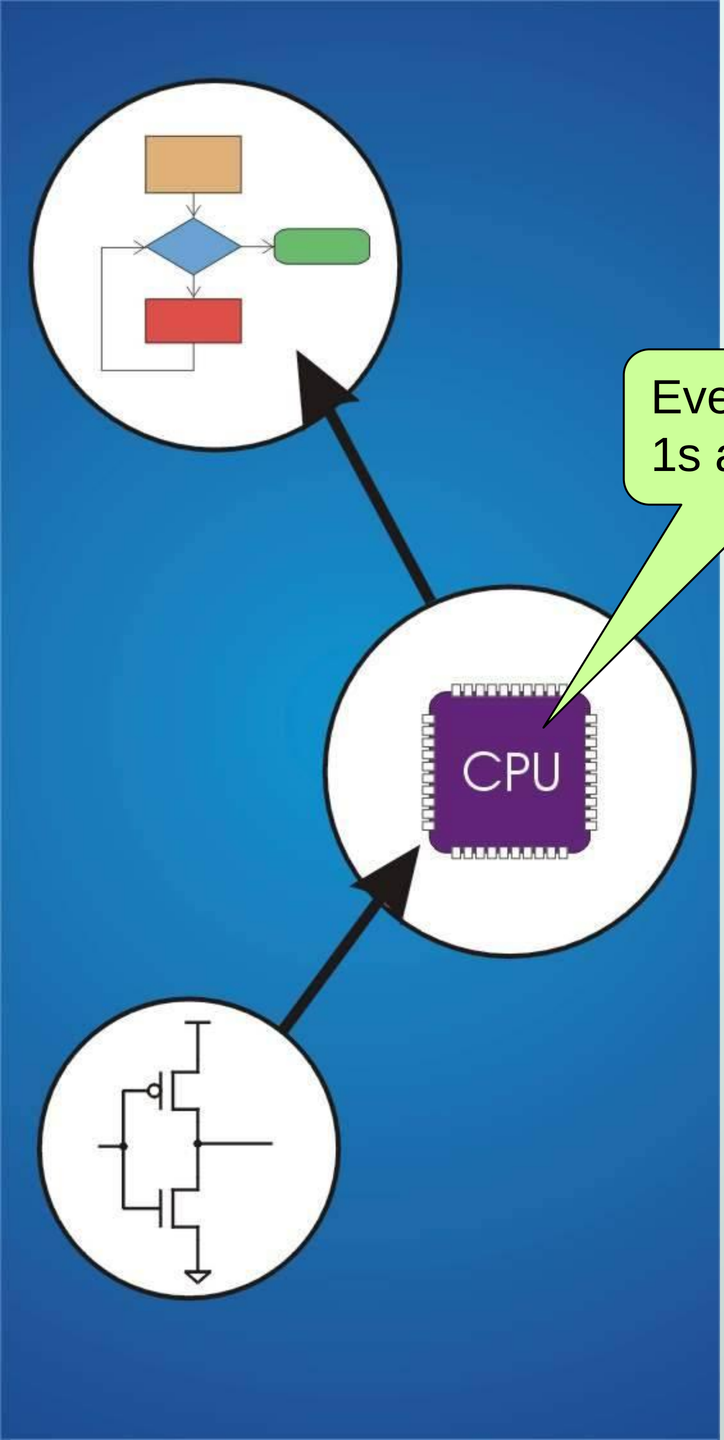
Announcements

- Welcome back.
- Assignment 0 (introduce yourself) is due tonight.
- Quiz 1 will be posted on RamCT soon. Due Sunday night after a week.
- HW1 will be posted RamCT later today. Will be due in a week on Thursday.
- Photo ID requirement for midterm and final.



Everything is 1s and 0s

Leibniz 1703

Gottfried Wilhelm Leibniz. Nach dem Stich von Etienne Ficquet (1731—94), wiedergegeben in W. v. Seidlitz, „Historisches Porträtwerk". Vgl. Text, S. 65.

Chapter 2
Bits, Data Types, and Operations

Original slides from Gregory Byrd, North Carolina State University

Modified slides by Chris Wilcox, Yashwant Malaiya

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How do we represent data in a computer?

- At the lowest level, a computer is an electronic machine.
  - works by controlling the flow of electrons
- Easy to recognize two conditions:
  1. Higher voltage – we'll call this state "1"
  2. Lower voltage – we'll call this state "0"
- Control
  - Turning transistors on or off
  - Like a light switch to

Computer is a binary digital system.

Digital system:
- finite number of symbols

Binary (base two) system:
- has two states: 0 and 1

- Basic unit of information is the *binary digit*, or ***bit***.
- Values with >2 states require multiple bits.
  - A collection of two bits has four possible states: 00, 01, 10, 11
  - A collection of three bits has eight possible states: 000, 001, 010, 011, 100, 101, 110, 111
  - *A collection of n bits has 2^n possible states.*

What kinds of data do we need to represent?

- Numbers – signed, unsigned, integers, floating point, complex, rational, irrational, …
- Text – characters, strings, …
- Instructions
- Logical – true, false
- Media
  - Images – pixels, colors, shapes, …
  - Sound – wave forms
- Data type:
  - *representation* and *operations* within the computer
- We'll start with numbers…

Unsigned Integers

- Binary numbers are just like decimal
  - Except there are only two digits (0, 1) instead 10 (0, 1, 2, ..9)
- Weighted positional notation
  - like decimal numbers: “329”
  - “3” is worth 300, because of its position, while “9” is only worth 9

329

10^2 10^1 10^0

3x100 + 2x10 + 9x1 = 329

most significant 101 *least significant bit*

2^2 2^1 2^0

1x4 + 0x2 + 1x1 = 5

Unsigned Integers (cont.)

- An *n*-bit unsigned integer represents 2^n values: from 0 to 2^n-1.

2^2	2^1	2^0	Decimal
0	0	0	0
0	0	1	1
0	1	0	2
0	1	1	3
1	0	0	4
1	0	1	5
1	1	0	6
1	1	1	7

Unsigned Binary Arithmetic

- Base-2 addition – just like base-10!
  - add from right to left, propagating carry

carry

```
  10010          10010          1111
+  1001        +  1011        +    1
  11011          11101         10000
```

```
  10111
+   111
```

Subtraction, multiplication, division,…

Signed Integers

- With n bits, we have 2^n distinct values.
  - assign about half to positive integers (1 through 2^{n-1})
  - assign about half to negative (- 2^{n-1} through -1)
  - that leaves two values: one for 0, and one extra
- Positive integers
  - just like unsigned – zero in *most significant* (MS) bit
    00101 = 5
- Negative integers
  - sign-magnitude – set sign bit to show negative
    10101 = -5
  - one's complement – flip every bit to represent negative
    11010 = -5
  - in either case, MS bit indicates sign: 0=pos., 1=neg.

Not good

Not good

Two's Complement

- Problems with sign-magnitude, 1's complement
  - two representations of zero (+0 and –0)
  - arithmetic circuits are complex
    - How to add two sign-magnitude numbers?
      - e.g., try 2 + (-3)
    - How to add to one's complement numbers?
      - e.g., try 4 + (-3)
- Solution: Two's complement

Used practically universally

Two's Complement

- ***Two's complement*** representation developed to make circuits easy for arithmetic.
  - for each positive number (X), assign value to its negative (-X),
    such that X + (-X) = 0 with "normal" addition, ignoring final carry out

```
  00101   (5)          01001   (9)
+ 11011   (-5)       + _____   (-9)
  00000   (0)          00000   (0)
```

Two's Complement Representation

- If number is positive or zero,
  - normal binary representation, zeroes in upper bit(s)
- If number is negative,
  - start with positive number
  - flip every bit (i.e., take the one's complement)
  - then add one

```
  00101  (5)              01001  (9)
  11010  (1's comp)              (1's comp)
+     1                 +     1
  11011  (-5)                    (-9)
```

Two's Complement Shortcut

- To take the two's complement of a number:
  - copy bits from right to left until (and including) first "1"
  - flip remaining bits to the left

```
  011010000
  100101111   (1's comp)
+         1
  ---------
  100110000
```

```
011010000
(flip)   (copy)
100110000
```

Two's Complement Signed Integers

- MS bit is sign bit – it has weight -2^{n-1}.
- Range of an n-bit number: -2^{n-1} through $2^{n-1} - 1$.
  - The most negative number has no positive counterpart.

-2^3	2^2	2^1	2^0	
0	0	0	0	0
0	0	0	1	1
0	0	1	0	2
0	0	1	1	3
0	1	0	0	4
0	1	0	1	5
0	1	1	0	6
0	1	1	1	7

-2^3	2^2	2^1	2^0	
1	0	0	0	-8
1	0	0	1	-7
1	0	1	0	-6
1	0	1	1	-5
1	1	0	0	-4
1	1	0	1	-3
1	1	1	0	-2
1	1	1	1	-1

smallest

Converting Binary (2's C) to Decimal

Remember this

1. If leading bit is one, take two's complement to get a positive number.
2. Add powers of 2 that have "1" in the corresponding bit positions.
3. If original number was negative, add a minus sign.

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024

$$X = 01101000_{two}$$
$$= 2^6+2^5+2^3 = 64+32+8$$
$$= 104_{ten}$$

Assuming 8-bit 2's complement numbers.

More Examples

$$X = 00100111_{two}$$
$$= 2^5+2^2+2^1+2^0 = 32+4+2+1$$
$$= 39_{ten}$$

$$X = 11100110_{two}$$
$$-X = 00011010$$
$$= 2^4+2^3+2^1 = 16+8+2$$
$$= 26_{ten}$$
$$X = -26_{ten}$$

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024

Assuming 8-bit 2's complement numbers.

Converting Decimal to Binary (2's C)

- First Method: ***Division***

1. Find magnitude of decimal number
2. Divide by two – remainder is least significant bit.
3. Keep dividing by two until answer is zero, writing remainders from right to left.
4. Append a zero as the MS bit; for negative, take two's complement.

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024

$X = 104_{ten}$

104 div 2	=	52	r 0	*bit 0*
52 div 2	=	26	r 0	*bit 1*
26 div 2	=	13	r 0	*bit 2*
13 div 2	=	6	r 1	*bit 3*
6 div 2	=	3	r 0	*bit 4*
3 div 2	=	1	r 1	*bit 5*
1 div 2	=	0	r 1	*bit 6*

$X = 01101000_{two}$

Converting Decimal to Binary (2's C)

- Second Method: ***Subtract Powers of Two***

1. Find magnitude of decimal number.
2. Subtract largest power of two less than or equal to number.
3. Put a one in the corresponding bit position.
4. Keep subtracting until result is zero.
5. Append a zero as MS bit; if original was negative, take two's complement.

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024

$X = 104_{ten}$

104 - 64	=	40	*bit 6*
40 - 32	=	8	*bit 5*
8 - 8	=	0	*bit 3*

$X = 01101000_{two}$

Operations: Arithmetic and Logical

- Recall: data types include *representation* and *operations*.
- 2's complement is a good representation for signed integers, now we need arithmetic operations:
  - Addition (including overflow)
  - Subtraction
  - Sign Extension
- Multiplication and division can be built from these basic operations.
- Logical operations are also useful:
  - AND
  - OR
  - NOT

Will see them soon

Addition

- As we've discussed, 2's comp. addition is just binary addition.
  - assume all integers have the same number of bits
  - ignore carry out
  - for now, assume that sum fits in n-bit 2's comp. representation

```
   01101000  (104)      11110110  (-10)
 + 11110000  (-16)    + ________  (-9)
   01011000  (88)                 (-19)
```

Assuming 8-bit 2's complement numbers.

Subtraction

- Negate subtrahend (2nd no.) and add.
  - assume all integers have the same number of bits
  - ignore carry out
  - for now, assume that difference fits in n-bit 2's comp. representation

	01101000	(104)	11110110	(-10)
-	00010000	(16)	-	(-9)
	01101000	(104)	11110110	(-10)
+	11110000	(-16)	+	(9)
	01011000	(88)		(-1)

Assuming 8-bit 2's complement numbers.

Sign Extension

- To add two numbers, we must represent them with the same number of bits.
- If we just pad with zeroes on the left:

4-bit	**8-bit**
`0100` (4)	`00000100` (still 4)
`1100` (-4)	`00001100` (12, not -4)

- Instead, replicate the MS bit -- the sign bit:

4-bit	**8-bit**
`0100` (4)	`00000100` (still 4)
`1100` (-4)	`11111100` (still -4)

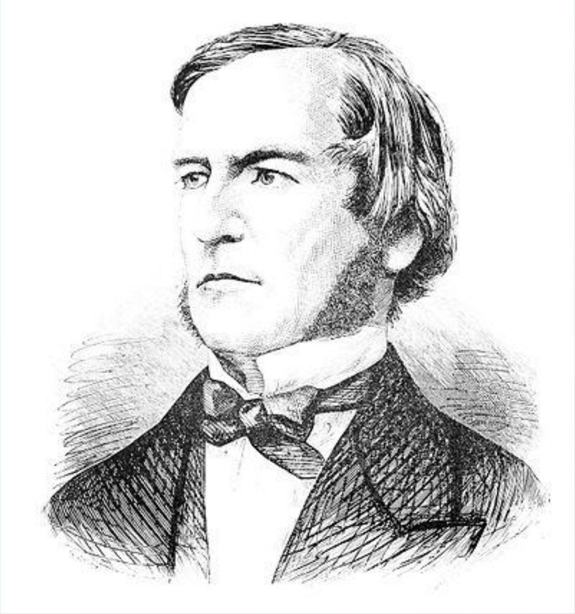
Overflow

- If operands are too big, then sum cannot be represented as an *n*-bit 2's comp number.

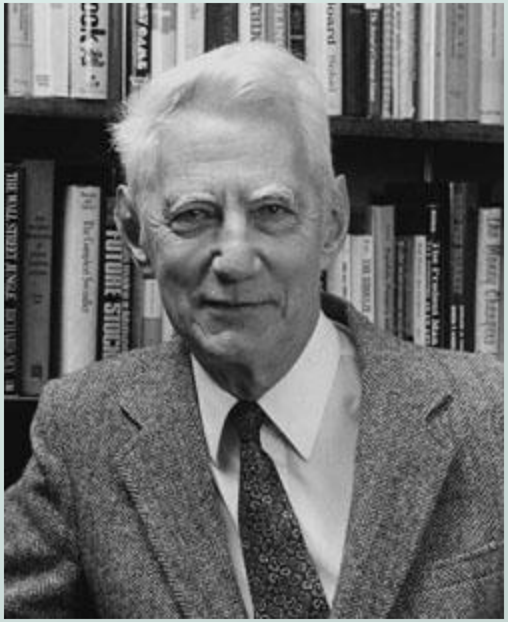
`01000`	(8)	`11000`	(-8)
+ `01001`	(9)	+ `10111`	(-9)
`10001`	(-15 ?)	`01111`	(+15)

- We have overflow if:
  - signs of both operands are the same, and
  - sign of sum is different.
- Another test -- easy for hardware:
  - carry into MS bit does not equal carry out

Logic Operations



George Boole (1815-1864)



Claude Shannon (1916-2001)

Logical Operations

Java/C: &&, ||, !

- Operations on logical TRUE or FALSE
  - two states -- takes one bit to represent: TRUE=1, FALSE=0

A	B	A AND B
0	0	0
0	1	0
1	0	0
1	1	1

A	B	A OR B
0	0	0
0	1	1
1	0	1
1	1	1

A	NOT A
0	1
1	0

- View *n*-bit number as a collection of *n* logical values
  - operation applied to each bit independently

Logical bitwise Operations

Java/C: &, |, ~

- AND
  - useful for clearing bits
    - AND with zero = 0
    - AND with one = no change
- OR
  - useful for setting bits
    - OR with zero = no change
    - OR with one = 1
- NOT
  - unary operation -- one argument
  - flips every bit

```
      11000101
AND   00001111
      --------
      00000101

      11000101
OR    00001111
      --------
      11001111

NOT   11000101
      --------
      00111010
```

Hexadecimal Notation

- It is often convenient to write binary (base-2) numbers in hexadecimal (base-16) instead.
  - fewer digits - four bits per hex digit
  - less error prone - no long string of 1's and 0's

Binary	Hex	Decimal	Binary	Hex	Decimal
0000	0	0	1000	8	8
0001	1	1	1001	9	9
0010	2	2	1010	A	10
0011	3	3	1011	B	11
0100	4	4	1100	C	12
0101	5	5	1101	D	13
0110	6	6	1110	E	14
0111	7	7	1111	F	15

Converting from Binary to Hexadecimal

- Every four bits is a hex digit.
  - start grouping from right-hand side

```
011  1010  1000  1111  0100  1101
                 0111
 3    A     8     F     4     D     7
```

> *This is not a new machine representation, just a convenient way to write the number.*

Fractions: Fixed-Point

- How can we represent fractions?
  - Use a "binary point" to separate positive from negative powers of two -- just like "decimal point."
  - 2's comp addition and subtraction still work (if binary points are aligned)

$2^{-1} = 0.5$

$2^{-2} = 0.25$

$2^{-3} = 0.125$

```
  00101000.101 (40.625)
+ 11111110.110 (-1.25)
  00100111.011 (39.375)
```

Only internal use

No new operations -- same as integer arithmetic.

Floating-Point Numbers

- Large values: 6.023×10^{23} -- requires 79 bits
- Small values: 6.626×10^{-34} -- requires >110 bits
- Use equivalent of "scientific notation": $F \times 2^{E}$
- Must have F (*fraction*), E (*exponent*), and sign.
- IEEE 754 Floating-Point Standard (32-bits):

1b	*8b*	*23b*
S	Exponent	Fraction

Single precision

$$N = (-1)^{S} \times 1.\text{fraction} \times 2^{\text{exponent} - 127},\ 1 \le \text{exponent} \le 254$$

$$N = (-1)^{S} \times 0.\text{fraction} \times 2^{-126},\ \text{exponent} = 0$$

Floating Point Example

- Single-precision IEEE floating point number:
- $\underline{1}\ \underline{01111110}\ \underline{10000000000000000000000}$

sign exponent fraction

  - Sign is 1 – number is negative.
  - Exponent field is 01111110 = 126 (decimal).
  - Fraction implies 1.100000000000… = 1.5 (decimal).

Always 1 for normalized numbers

- Value = $-1.5 \times 2^{(126-127)} = -1.5 \times 2^{-1} = -0.75$

Floating-Point Operations

- Special cases: 0 (all zeros), infinity, etc.
- Will regular 2's complement arithmetic work for Floating Point numbers?
- (*Hint*: In decimal, how do we compute $3.07 \times 10^{12} + 9.11 \times 10^{8}$?)

Text: ASCII Characters

New line:
Unix: LF
Windows: LF+CR

- ASCII: Maps 128 characters to 7-bit code.
  - printable and non-printable (ESC, DEL, …) characters

00 nul	10 dle	20 sp	30 0	40 @	50 P	60 `	70 p
01 soh	11 dc1	21 !	31 1	41 A	51 Q	61 a	71 q
02 stx	12 dc2	22 "	32 2	42 B	52 R	62 b	72 r
03 etx	13 dc3	23 #	33 3	43 C	53 S	63 c	73 s
04 eot	14 dc4	24 $	34 4	44 D	54 T	64 d	74 t
05 enq	15 nak	25 %	35 5	45 E	55 U	65 e	75 u
06 ack	16 syn	26 &	36 6	46 F	56 V	66 f	76 v
07 bel	17 etb	27 '	37 7	47 G	57 W	67 g	77 w
08 bs	18 can	28 (	38 8	48 H	58 X	68 h	78 x
09 ht	19 em	29)	39 9	49 I	59 Y	69 i	79 y
0a lf	1a sub	2a *	3a :	4a J	5a Z	6a j	7a z
0b vt	1b esc	2b +	3b ;	4b K	5b [	6b k	7b {
0c np	1c fs	2c ,	3c <	4c L	5c \	6c l	7c |
0d cr	1d gs	2d -	3d =	4d M	5d]	6d m	7d }
0e so	1e rs	2e .	3e >	4e N	5e ^	6e n	7e ~
0f si	1f us	2f /	3f ?	4f O	5f _	6f o	7f del

Interesting Properties of ASCII Code

- What is relationship between a decimal digit ('0', '1', …) and its ASCII code?
- What is the difference between an upper-case letter ('A', 'B', …) and its lower-case equivalent ('a', 'b', …)?
- Given two ASCII characters, how do we tell which comes first in alphabetical order?
- Are 128 characters enough? (http://www.unicode.org/)

No new operations -- integer arithmetic and logic.

Other Data Types

- Text strings
  - sequence of characters, terminated with NULL (0)
  - typically, no hardware support
- Image: several formats
  - array of pixels
    - monochrome: one bit (1/0 = black/white)
    - color: red, green, blue (RGB) components
    - other properties: transparency
  - hardware support:
    - typically none, in older general-purpose processors
    - MMX -- multiple 8-bit operations on 32-bit word
- Sound, video
  - Several file formats

LC-3 Data Types

It is a simple processor

- Some data types are supported directly by the instruction set architecture.
- For LC-3, there is only one hardware-supported data type:
  - 16-bit 2's complement signed integer
  - Operations: ADD, AND, NOT
- Other data types are supported by interpreting 16-bit values as logical, text, fixed-point, etc., in the software that we write.