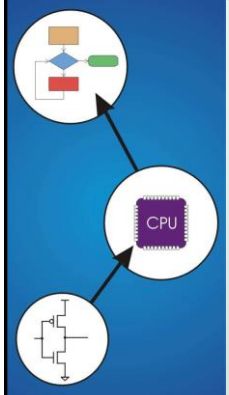


Chapter 17
Recursion

Original slides from Gregory Byrd, North Carolina State University

Modified slides by Chris Wilcox, Colorado State University

What is Recursion?

- A **recursive function** is one that solves its task by **calling itself** on smaller pieces of data.
  - Similar to recurrence function in mathematics.
  - Like iteration -- can be used interchangeably; sometimes recursion results in a simpler solution.

Example: Running sum ($\sum_{1}^{n} i$)

Mathematical Definition:

RunningSum(1) = 1
RunningSum(n) =
n + RunningSum(n-1)

Recursive Function:

```
int RunningSum(int n) {
  if (n == 1)
    return 1;
  else
    return n + RunningSum(n-1);
}
```


Executing RunningSum

```
res = RunningSum(4);
```
return value = 10 — RunningSum(4)
```
return 4 + RunningSum(3);
```
return value = 6 — RunningSum(3)
```
return 3 + RunningSum(2);
```
return value = 3 — RunningSum(2)
```
return 2 + RunningSum(1);
```
return value = 1 — RunningSum(1)
```
return 1;
```


High-Level Example: Binary Search

- Given a sorted set of exams, in alphabetical order, find the exam for a particular student.

1. Look at the exam **halfway** through the pile.
2. If it matches the name, we're done;
   if it does not match, then...

3a. If the name is greater (alphabetically), then **search the upper half** of the stack.

3b. If the name is less than the halfway point, then **search the lower half** of the stack.

Binary Search: Pseudocode

- Pseudocode is a way to describe algorithms without completely coding them in C.

```
FindExam(studentName, start, end) {
  halfwayPoint = (end + start)/2;
  if (end < start)
    ExamNotFound();  /* exam not in stack */
  else if (studentName == NameOfExam(halfwayPoint))
    ExamFound(halfwayPoint); /* found exam! */
  else if (studentName < NameOfExam(halfwayPoint))
    /* search lower half */
    FindExam(studentName, start, halfwayPoint-1)
  else
    /* search upper half */
    FindExam(studentName, halfwayPoint + 1, end);
}
```


Detailed Example: Fibonacci Numbers

- Mathematical Definition:

$$f(n) = f(n-1) + f(n-2)$$
$$f(1) = 1$$
$$f(0) = 1$$

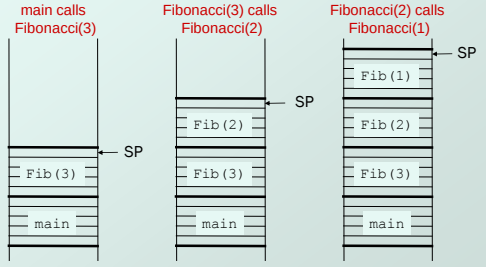
- In other words, the n-th Fibonacci number is the sum of the previous two Fibonacci numbers.

Fibonacci: C Code

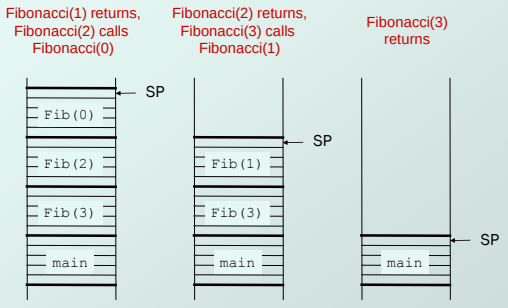
```
int Fibonacci(int n)
{
  if ((n == 0) || (n == 1))
    return 1;
  else
    return Fibonacci(n-1) + Fibonacci(n-2);
}
```


Activation Records

- Whenever Fibonacci is invoked, a new activation record is pushed onto the stack.



Activation Records (cont.)



Tracing the Function Calls

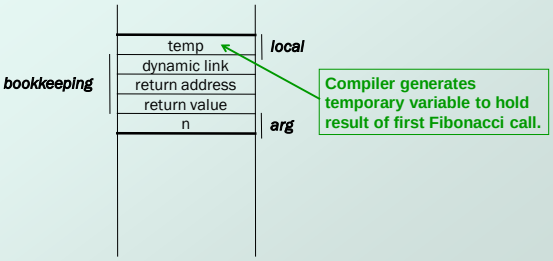
- If we are debugging this program, we might want to trace all the calls of Fibonacci.
  - Note: A trace will also contain the arguments passed into the function.
- For Fibonacci(3), a trace looks like:

```
Fibonacci(3)
Fibonacci(2)
Fibonacci(1)
Fibonacci(0)
Fibonacci(1)
```

- What would trace of Fibonacci(4) look like?

Fibonacci: LC-3 Code

Activation Record



A Final C Example: Printing an Integer

- Recursively converts an unsigned integer as a string of ASCII characters.
  - If integer <10, convert to char and print.
  - else, call self on first (n-1) digits and then print last digit.

```
void IntToAscii(int num) {
  int prefix, currDigit;
  if (num < 10)
    putchar(num + '0');    /* print number */
  else {
    prefix = num / 10;     /* previous digits */
    digit = num % 10;      /* current digit */
    IntToAscii(prefix);    /* recursive call */
    putchar(digit + '0'); /* print digit */
  }
}
```

Trace of IntToAscii

- Calling IntToAscii with parameter 12345:

```
IntToAscii(12345)
 IntToAscii(1234)
  IntToAscii(123)
   IntToAscii(12)
    IntToAscii(1)
    putchar('1')
   putchar('2')
  putchar('3')
 putchar('4')
putchar('5')
```