

Counting: Basics

Rosen, Chapter 6.1

A simple counting problem

- You have 6 pairs of pants and 10 shirts. How many different outfits does this give?
- Possible answers:
 - A) 6×10
 - B) $6 + 10$

Counting: the product rule

- If there are n_1 ways of doing one task, and for each of those there are n_2 ways of doing a second task, then there are $n_1 n_2$ ways of performing both tasks.
- Example:
 - You have 6 pairs of pants and 10 shirts. How many different outfits does this give?

Relation to Cartesian products

- You have 6 pairs of pants and 10 shirts. How many different outfits does this give?
- We can express the set of all outfits as:
 $\{(s, p) \mid s \in \text{shirts and } p \in \text{pants}\}$

Relation to Cartesian products

- You have 6 pairs of pants and 10 shirts. How many different outfits does this give?
- We can express the set of all outfits as:
 $\{(s, p) \mid s \in \text{shirts and } p \in \text{pants}\}$
- This is an example of a Cartesian product

Relation to Cartesian products

- The **Cartesian product** of sets A and B is denoted by $A \times B$ and is defined as:
$$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$$
- $|A \times B| = |A| \cdot |B|$

How would a program for enumerating outfits work?

- You would like to print all elements of the set $\{(s, p) \mid s \in \text{shirts and } p \in \text{pants}\}$
Assume that shirts and pants are stored in two arrays.

Product rule

- Colorado assigns license plate as three digits followed by three uppercase letters. How many license plates are possible?



Product rule

- Colorado assigns license plates as three digits followed by three uppercase letters. How many license plates are possible?

- A) $3^{10} \times 3^{26}$
- B) $2 \times 3^{10} \times 3^{26}$
- C) $10^3 \times 26^3$

More examples

- How many bit strings with 7 digits are there?
- How many functions are there from a set with m elements to a set with n elements?
- A) m^n B) n^m
- How many one-to-one functions are there from a set with m elements to a set with n elements? (Rosen Section 2.3)

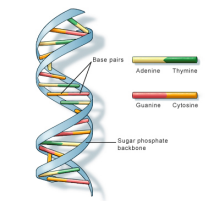
More examples

- Use the product rule to show that the number of different subsets of a finite set S is $2^{|S|}$
- Product rule and Cartesian products:
 - $|A_1 \times A_2 \times \dots \times A_n| = |A_1| \cdot |A_2| \cdot \dots \cdot |A_n|$

DNA and proteins

- DNA is a long chain of nucleotides, each one being A, C, G, or T. DNA can code for proteins that are chains of amino acids. There are 20 amino acids. How many nucleotides does it take to code for a single amino acid?

- A) 2
- B) 3
- C) 4



A different counting problem

- X has decided to shop at a **single** store, either in old town or the foothills mall. If X visits old town, X will shop at one of three stores. If X visits the mall, then X will shop at one of two stores. How many ways could X end up shopping?

- A) $3 + 2$
- B) 3×2
- C) $3! \times 2!$

The Sum Rule

- If a task can be done either in one of n_1 ways **or** in one of n_2 ways, and none of the n_1 ways is the same as the n_2 ways, then there are $n_1 + n_2$ ways to do the task.
- This is a statement about set theory: if two sets A and B are disjoint then

$$|A \cup B| = |A| + |B|$$

Example

- A student can choose a computer project from one of three lists. The three lists contain 23, 15, and 4 possible projects. No project is on more than one list. How many possible projects are there to choose from?

Example

- How many license plates can be made using either two or three uppercase letters followed by two or three digits?

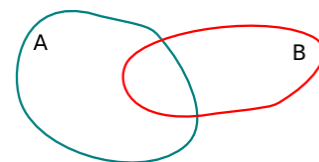
Example

- Suppose you need to pick a password that has 6, 7 or 8 characters, where each character is an uppercase letter or a digit, and each password must contain at least one digit. How many possible passwords are there?
- Hint: need to use both the product rule and the sum rule.

The inclusion exclusion principle

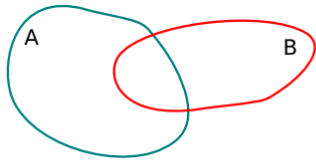
- A more general statement than the sum rule:

$$|A \cup B| = |A| + |B| - |A \cap B|$$



Example

- How many numbers between 1 and 30 are divisible by 2 or 3?



The inclusion exclusion principle

- How many bit strings of length eight start with a 1 **or** end with 00?

1 - - - - - how many?
- - - - - 0 0 how many?

if I add these, how many have I counted twice?

A hairy problem

- In Denver there are two people that have the same number of hairs.

- A) True
- B) False

On average, non balding people have 90K-150K hairs depending on color. So let's assume the maximum number is less than 300K.

The pigeonhole principle

- If k is a positive integer and $k+1$ or more objects are placed into k boxes, then there is at least one box containing two or more objects.



Image: <http://en.wikipedia.org/wiki/File:TooManyPigeons.jpg>

Examples

- In a group of 367 people, there must be at least two with the same birthday
- A drawer contains a dozen brown socks and a dozen black socks, all unmatched. A guy takes socks out at random in the dark.
 - How many socks must he take out to be sure that he has at least two socks of the same color?A) 13 B) 3 C) 12

Examples

- In a group of 367 people, there must be at least two with the same birthday
- A drawer contains a dozen brown socks and a dozen black socks, all unmatched. A guy takes socks out at random in the dark.
 - How many socks must he take out to be sure that he has at least two socks of the same color?
 - How many socks must he take out to be sure that he has at least two black socks?

Examples

- Show that if five different digits between 1 and 8 are selected, there must be at least one pair of these with a sum equal to 9.
- ask yourself: what are the pigeon holes?
what are the pigeons?