Return to Chuck

CS161 Lab Week 9: Counting

In this lab you will practice solving counting problems. Write your answers to the following problems on paper and show the instructor your answers before leaving. The instructor will lead initial discussion of each question.

1. How many 6 character passwords are there that only use lowercase letters?
2. How many 6 character passwords are there that start with 2 digits and end with 4 lowercase letters?
3. How many 5 or 6 character passwords are there that use only lowercase letters?
4. How many people do you need in a room before there are [at least] two people that were born in the same month?
5. How many ways are there for a person to have 3 initials?
6. How many bit strings are there of length six or less?
7. How many bit strings of length seven begin with two 0s AND end with three 1s?
8. How many bit strings of length seven either begin with two 0s OR end with three 1s?
9. How many people do you need in a room before you can guarantee that two people have names that start with the same letter?
10. How many people do you need in a room before you can guarantee that [at least] two people have the same birthday?
11. How many different words of length 8 can you form in the English language such that the first letter is the same as the last letter?
12. How many different words of length 8 can you form in the English language that contain exactly one vowel?
13. How many different words of length 8 can you form in the English language that contain at least one vowel?
14. How many 6-character lowercase passwords are there, that begin with 'r' or end with 't'?
15. How many cards must you draw before you are guaranteed to have two of the same suit?
16. How many people do you need in a room before you can guarantee that there are four from the same state?
17. How many cards must you draw before you are guaranteed to have a spade?
18. How many cards must you draw before you are guaranteed to have two kings?

$\frac{2}{14}$ $\frac{3}{25}$ $\frac{4}{40}$

$\frac{13}{39}$

1. 6 character passwords of lowercase letters.

a…z each slot:

$$26 \cdot 26 \cdot 26 \cdot 26 \cdot 26 \cdot 26 = 26^6$$

Product Rule

2. 6 character passwords, start with 2 digits and end with 4 lowercase letters

0…9, 0…9, a…z, a…z, a…z, a…z

$$10 \cdot 10 \cdot 26 \cdot 26 \cdot 26 \cdot 26 = 10^2\, 26^4$$

Product Rule

3. 5 or 6 character passwords of lowercase letters.

a…z each slot:

$$26 \cdot 26 \cdot 26 \cdot 26 \cdot 26 = 26^5$$ Product Rule

OR + Sum Rule

$$26 \cdot 26 \cdot 26 \cdot 26 \cdot 26 \cdot 26 = 26^6$$ Product Rule

$$26^5 + 26^6$$

4. How many people must be in a room to guarantee there are at least two people who were born in the same month?

13

Pigeon-Hole Principle

5. How many ways are there for a person to have 3 initials? Upper Case

A…Z each slot:

$$26 \cdot 26 \cdot 26 = 26^3$$

Product Rule

(6) How bit strings are there of length 6 or less?

$$\frac{0,1}{2}\cdot\frac{0,1}{2}\cdot\frac{0,1}{2}\cdot\frac{0,1}{2}\cdot\frac{0,1}{2}\cdot\frac{0,1}{2} = 2^6 \quad \text{Product Rule}$$

$+$ Sum Rule

$$\frac{0,1}{2}\;\frac{0,1}{2}\;\frac{0,1}{2}\;\frac{0,1}{2}\;\frac{0,1}{2} = 2^5 \quad \text{Product Rule}$$

$+$ Sum Rule

$\vdots$

$+$ Sum Rule

$$\frac{0,1}{2} \qquad 2$$

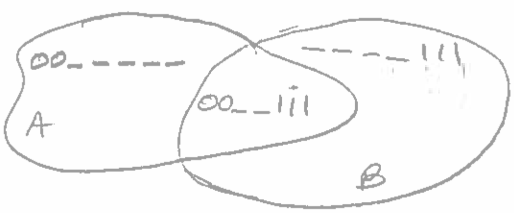
$$2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2$$

$$= \sum_{i=1}^{6} 2^i$$

(7) bit strings length 7, begin with 2 ϕ's AND end with 3 1's.

$$\frac{0}{1}\cdot\frac{0}{1}\cdot\frac{0,1}{2}\cdot\frac{0,1}{2}\cdot\frac{1}{1}\cdot\frac{1}{1}\cdot\frac{1}{1} = 2^2 \quad \text{Product Rule}$$

(8) bit strings length 7, begin with 2 ϕ's OR end with 3 1's.



$$|A| + |B| - |A \cap B|$$

$|A|$

$$\frac{0}{1}\cdot\frac{0}{1}\cdot\frac{0,1}{2}\cdot\frac{0,1}{2}\cdot\frac{0,1}{2}\cdot\frac{0,1}{2}\cdot\frac{0,1}{2} = 2^5$$

$$2^5 + 2^4 - 2^2$$

$|B|$

$$\frac{0,1}{2}\cdot\frac{0,1}{2}\cdot\frac{0,1}{2}\cdot\frac{0,1}{2}\cdot\frac{1}{1}\cdot\frac{1}{1}\cdot\frac{1}{1} = 2^4$$

$|A \cap B|$ already found in Problem (8) 2^2

(9). Minimum number of people to guarantee that at least two have names that start with same letter.

26 letters — 27 people — Pigeon Hole Principle

(10) Minimum number of people to guarantee that at least two have the same birthday. Non-leap year

365 days (Pigeon Holes)

366 people (Pigeons) — Pigeon Hole Principle

(11) How many words of length 8 can you form in the English language such that the first letter is the same as the last letter? (ignore case)

a ... z for each of the first 7 positions; last position: Determined by first letter

$$\underline{26}\cdot\underline{26}\cdot\underline{26}\cdot\underline{26}\cdot\underline{26}\cdot\underline{26}\cdot\underline{26}\cdot\underline{1} = 26^7$$

Product Rule

(12) words length 8 with exactly one vowel?

8 ways to pick location of vowel, leaving 7 positions to place a consonant. There are 5 vowels and $26-5=21$ consonant.

$$(8\cdot 5)\cdot 21^7$$

vowel — consonants

(13) words length 8 with at least one vowel.

|all words| − |words with no vowels|

$$= 26^8 - 21^8$$

Inclusion-Exclusion Rule

(14) 6-character lowercase (all letters) passwords that begin with 'r' or end with 't'

r $\frac{z}{26}\cdot\frac{z}{26}\cdot\frac{z}{26}\cdot\frac{z}{26}\cdot\frac{z}{26} = 26^5$

$\frac{z}{26}\cdot\frac{z}{26}\cdot\frac{z}{26}\cdot\frac{z}{26}\cdot\frac{z}{26}$ t $= 26^5$

r $\frac{z}{26}$ – – – t 26^4

$26^5 + 26^5 - 26^4$

(15) Minimum number of cards must you draw to guarantee that you have two of the same suit?

4 suits (Pigeon Holes) — Pigeon Hole Principle

5 cards (Pigeons)

(16) Minimum number of people to guarantee there are four from the same state.

50 states (Pigeon Holes) — Pigeon Hole Principle

$(4-1)50+1 =$ 151 people (Pigeons)

(17) Minimum number of cards to guarantee at least one spade

Worst case is drawing all cards of the other 3 suits first. There are 13 cards in each suit. So you must draw $13 \cdot 3 + 1 =$ 40 cards

(18) Minimum number of cards to guarantee at least 2 kings

Worst case is all 4 kings are at end of deck. A deck has 52 cards, so you must draw 50 cards

8') Minimum number of cards to guarantee at least 2 cards of same value

13 card values (Pigeon Holes)

14 cards (Pigeons)

How about 3 of same value?

$13(3-1)+1 = 25$ cards