CS 301 - Lecture 10
Chomsky and Greibach Normal Forms

Fall 2008

Review

- Languages and Grammars
  - Alphabets, strings, languages
- Regular Languages
  - Deterministic Finite and Nondeterministic Automata
  - Equivalence of NFA and DFA
  - Regular Expressions
  - Regular Grammars
  - Properties of Regular Languages
  - Languages that are not regular and the pumping lemma
- Context Free Languages
  - Context Free Grammars
  - Derivations: leftmost, rightmost and derivation trees
  - Parsing and ambiguity
  - Simplifying Context Free Grammars
- Today:
  - More Simplifications
  - Normal Forms

Nullable Variables

$\lambda -$production: $A \to \lambda$

Nullable Variable: $A \Rightarrow \ldots \Rightarrow \lambda$

Which Variables are Nullable?

0) Nullable Variables = $V_n = \emptyset$

For every production $A \rightarrow \lambda$

Add A to V_n

1) For every variable $B \notin V_n$

check each production $B \rightarrow A_1 A_2 ... A_n$

Add B to V_n if all $A_i \in V_n$

2) If step 1 added any B to V_n

repeat step 1

Removing Nullable Variables

Example Grammar:

$$S \to aMb$$
$$M \to aMb$$
$$M \to \lambda$$

Nullable variable

$$S \to aMb$$
$$M \to aMb$$
$$\cancel{M \to \lambda}$$

Substitute
$M \to \lambda$

Final Grammar

$$S \to aMb$$
$$S \to ab$$
$$M \to aMb$$
$$M \to ab$$

Unit-Productions

Unit Production: $A \to B$

(a single variable in both sides)

Removing Unit Productions

Observation:

$$A \to A$$

Is removed immediately

Example Grammar:

$S \to aA$
$A \to a$
$A \to B$
$B \to A$
$B \to bb$

$S \to aA$
$A \to a$
$\cancel{A \to B}$
$B \to A$
$B \to bb$

Substitute
$A \to B$

$S \to aA \mid aB$
$A \to a$
$B \to A \mid B$
$B \to bb$

$S \to aA \mid aB$
$A \to a$
$B \to A \mid \cancel{B}$
$B \to bb$

Remove
$B \to B$

$S \to aA \mid aB$
$A \to a$
$B \to A$
$B \to bb$

$S \to aA \mid aB$
$A \to a$
$\cancel{B \to A}$
$B \to bb$

Substitute
$B \to A$

$S \to aA \mid aB \mid aA$
$A \to a$
$B \to bb$

Remove repeated productions

$$S \to aA \mid aB \mid \cancel{aA}$$
$$A \to a$$
$$B \to bb$$

Final grammar

$$S \to aA \mid aB$$
$$A \to a$$
$$B \to bb$$

Removing All

- **Step 1:** Remove Nullable Variables
- **Step 2:** Remove Unit-Productions
- **Step 3:** Remove Useless Variables

Normal Forms
for
Context-free Grammars

Chomsky Normal Form

Each productions has form:

$A \to BC$ or $A \to a$

variable variable terminal

Examples:

$$S \to AS$$
$$S \to a$$
$$A \to SA$$
$$A \to b$$

Chomsky
Normal Form

$$S \to AS$$
$$S \to \boxed{AAS}$$
$$A \to SA$$
$$A \to \boxed{aa}$$

Not Chomsky
Normal Form

Conversion to Chomsky Normal Form

- Example:

$$S \to ABa$$
$$A \to aab$$
$$B \to Ac$$

Not Chomsky
Normal Form

Introduce variables for terminals: T_a, T_b, T_c

$$S \to ABa$$
$$A \to aab$$
$$B \to Ac$$

$\Rightarrow$

$$S \to ABT_a$$
$$A \to T_aT_aT_b$$
$$B \to AT_c$$
$$T_a \to a$$
$$T_b \to b$$
$$T_c \to c$$

Introduce intermediate variable: V_1

$$S \to ABT_a$$
$$A \to T_aT_aT_b$$
$$B \to AT_c$$
$$T_a \to a$$
$$T_b \to b$$
$$T_c \to c$$

$\Rightarrow$

$$S \to AV_1$$
$$V_1 \to BT_a$$
$$A \to T_aT_aT_b$$
$$B \to AT_c$$
$$T_a \to a$$
$$T_b \to b$$
$$T_c \to c$$

Introduce intermediate variable: V_2

$S \to AV_1$
$V_1 \to BT_a$
$A \to T_aT_aT_b$
$B \to AT_c$
$T_a \to a$
$T_b \to b$
$T_c \to c$

$\Longrightarrow$

$S \to AV_1$
$V_1 \to BT_a$
$A \to T_aV_2$
$V_2 \to T_aT_b$
$B \to AT_c$
$T_a \to a$
$T_b \to b$
$T_c \to c$

Final grammar in Chomsky Normal Form:

Initial grammar

$S \to ABa$
$A \to aab$
$B \to Ac$

$S \to AV_1$
$V_1 \to BT_a$
$A \to T_aV_2$
$V_2 \to T_aT_b$
$B \to AT_c$
$T_a \to a$
$T_b \to b$
$T_c \to c$

In general:

From any context-free grammar
(which doesn't produce λ)
not in Chomsky Normal Form

we can obtain:
An equivalent grammar
in Chomsky Normal Form

The Procedure

First remove:

Nullable variables

Unit productions

Then, for every symbol a:

Add production $T_a \rightarrow a$

In productions: replace a with T_a

New variable: T_a

Replace any production $A \rightarrow C_1 C_2 \cdots C_n$

with
$$A \rightarrow C_1 V_1$$
$$V_1 \rightarrow C_2 V_2$$
$$\dots$$
$$V_{n-2} \rightarrow C_{n-1} C_n$$

New intermediate variables: $V_1, V_2, \ldots, V_{n-2}$

Theorem: For any context-free grammar (which doesn't produce λ) there is an equivalent grammar in Chomsky Normal Form

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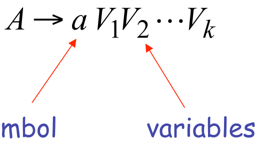
Observations

- Chomsky normal forms are good for parsing and proving theorems

- It is very easy to find the Chomsky normal form for any context-free grammar

Greibach Normal Form

All productions have form:



$$A \rightarrow a\, V_1 V_2 \cdots V_k \qquad k \geq 0$$

Examples:

$$S \rightarrow cAB$$
$$A \rightarrow aA \mid bB \mid b$$
$$B \rightarrow b$$

Greibach
Normal Form

$$S \rightarrow abSb$$
$$S \rightarrow aa$$

Not Greibach
Normal Form

Conversion to Greibach Normal Form:

$$S \rightarrow abSb \qquad S \rightarrow aa$$

$$\Rightarrow$$

$$S \rightarrow aT_bST_b$$
$$S \rightarrow aT_a$$
$$T_a \rightarrow a$$
$$T_b \rightarrow b$$

Greibach
Normal Form

Theorem: For any context-free grammar (which doesn't produce λ) there is an equivalent grammar in Greibach Normal Form

Observations

- Greibach normal forms are very good for parsing

- It is hard to find the Greibach normal form of any context-free grammar

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What's Next

- Read
  - Linz Chapter 1,2.1, 2.2, 2.3, (skip 2.4), 3, 4, 5, 6.1, 6.2, (skip 6.3), and 7.1
  - JFLAP Chapter 1, 2.1, (skip 2.2), 3, 4, 5, 6.1, 7
- Next Lecture Topics from Chapter 7.1
  - Nondeterminstic Pushdown Automata
- Quiz 2 in Recitation on Wednesday 10/1
  - Covers Linz 2, 3, 4 and JFLAP 3, 4
  - Closed book, but you may bring one sheet of 8.5 x 11 inch paper with any notes you like.
  - Quiz will take the full hour
- Homework
  - Homework Due Today
  - New Homework Available Friday Morning
  - New Homework Due Next Thursday