Minimization of Boolean Functions using Karnaugh Maps

Maurice Karnaugh 1953

Minimization

- Minimization can be done using
  - Boolean algebra
    - To combine terms

$$B\bar{C} + BC = B(\bar{C} + C) = B$$

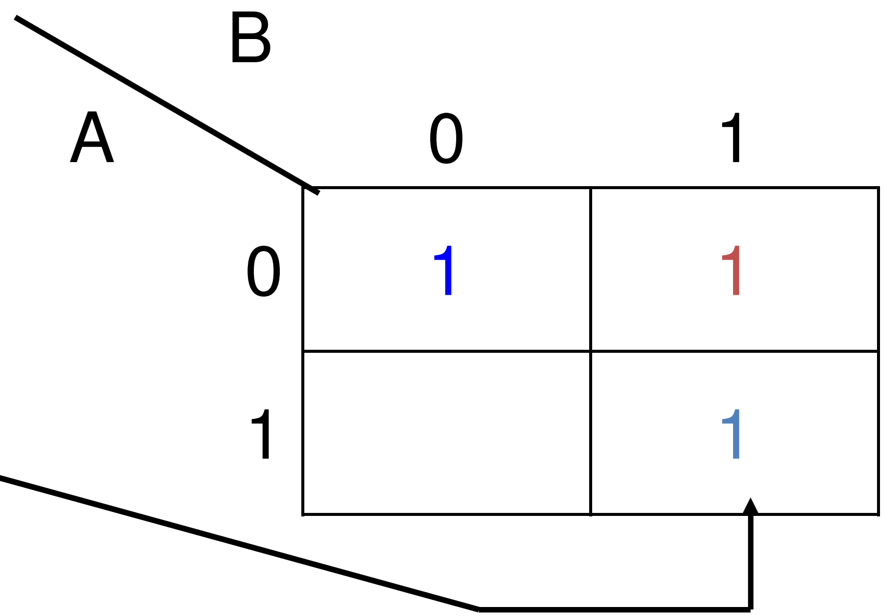
- Or equivalently
  - Karnaugh maps
    - Visual identification of terms that can be combined

Karnaugh Maps

- K-Maps are a convenient way to simplify Boolean Expressions.
- They can be used for up to 4 (or 5) variables.
- They are a visual representation of a truth table.
- Expression are most commonly expressed in sum of products form.

Truth table to K-Map

A	B	P
0	0	1
0	1	1
1	0	0
1	1	1



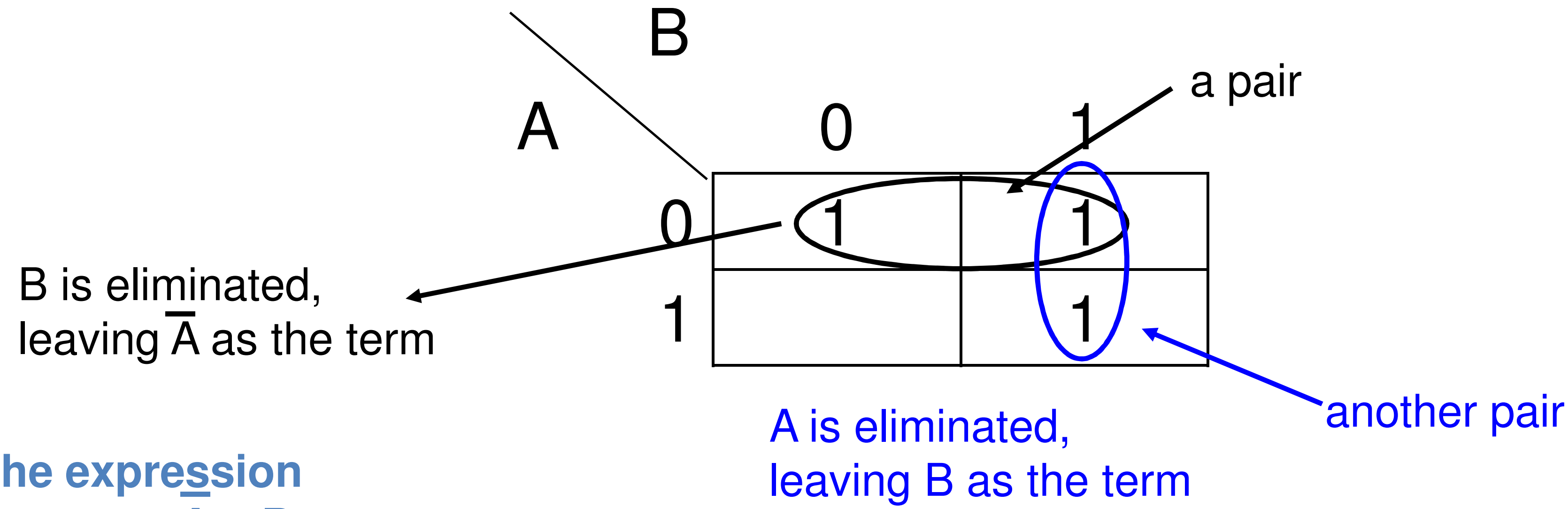
The expression is:

$$\overline{A}.\overline{B} + \overline{A}.B + A.B$$

minterms are represented by a 1 in the corresponding location in the K map.

K-Maps

- Adjacent 1’s can be “paired off”
- Any variable which is both a 1 and a zero in this pairing can be eliminated
- Pairs may be adjacent horizontally or vertically

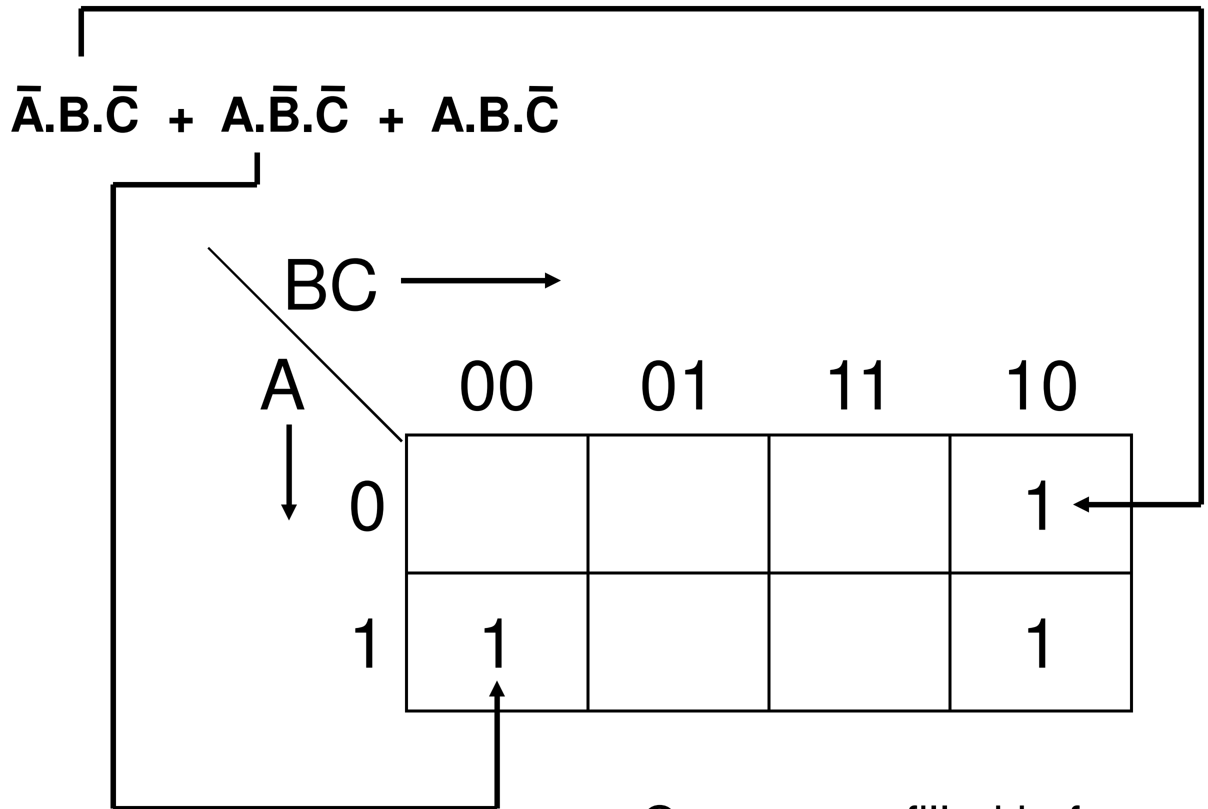


The expression becomes A + B

An example

- Two Variable K-Map

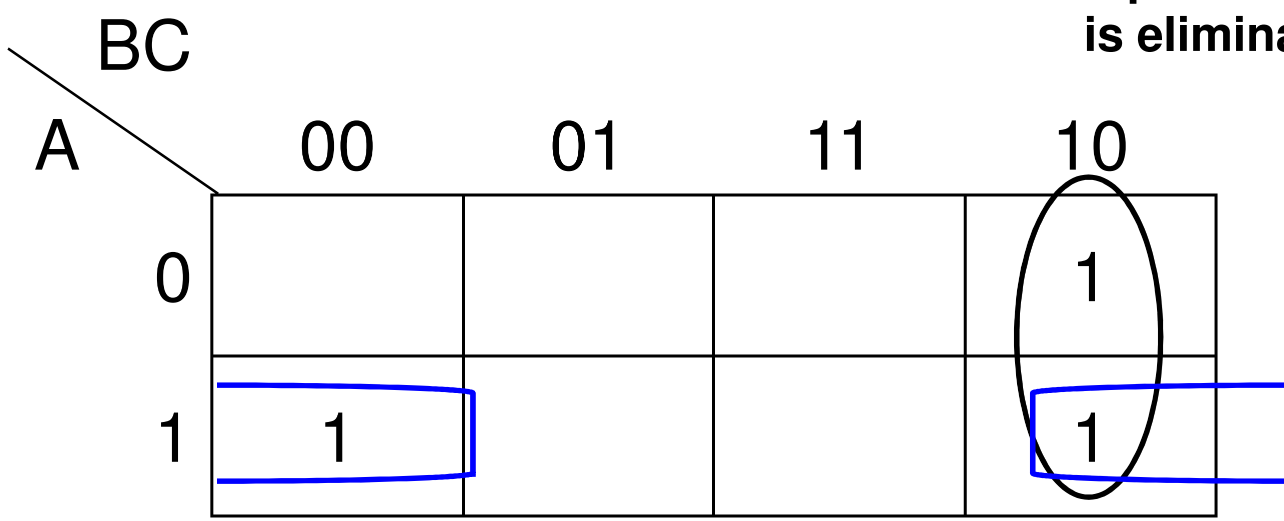
A	B	C	P
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0



One square filled in for each minterm.

Grouping the Pairs

equates to $B.\overline{C}$ as A is eliminated.



Our truth table simplifies to $A.\overline{C} + B.\overline{C}$

Here, we can "wrap around" and this pair equates to $A.\overline{C}$ as B is eliminated.

Groups of 4

Groups of 4 in a block can be used to eliminate two variables:

A \ BC	00	01	11	10
0			1	1
1			1	1

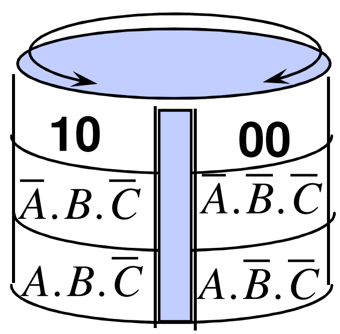
The solution is B because it is a 1 over the whole block

(vertical pairs) = $BC + B\overline{C} = B(C + \overline{C}) = B$.

Karnaugh Maps

- Three Variable K-Map

A \ BC	00	01	11	10
0	$\overline{A}.\overline{B}.\overline{C}$	$\overline{A}.\overline{B}.C$	$\overline{A}.B.C$	$\overline{A}.B.\overline{C}$
1	$A.\overline{B}.\overline{C}$	$A.\overline{B}.C$	$A.B.C$	$A.B.\overline{C}$



- Extreme ends of same row are *adjacent*

Karnaugh Maps

- Three Variable K-Map example

$$X = \overline{A}.\overline{B}.\overline{C} + A.\overline{B}.\overline{C} + \overline{A}.B.\overline{C} + A.B.\overline{C}$$

A \ BC	00	01	11	10
0				
1				

$X =$

The Block of 4, again

BC / A	00	01	11	10
0	1			1
1	1			1

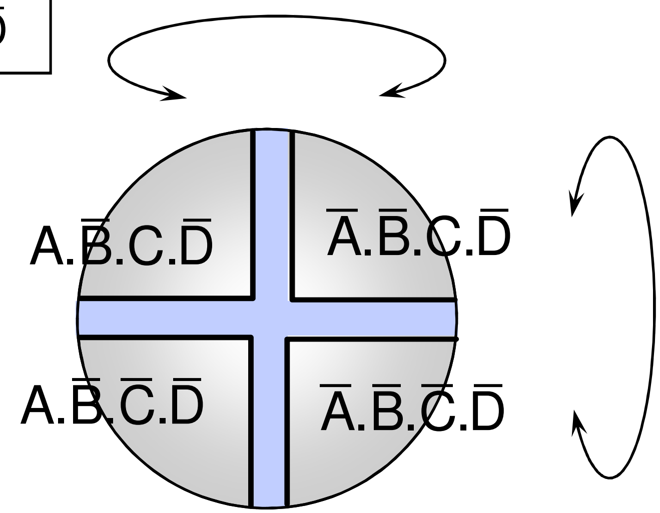
$X = \overline{C}$

4-variable Karnaugh Maps

- Four Variable K-Map

AB \ CD	00	01	11	10
00	$\overline{A}.\overline{B}.\overline{C}.\overline{D}$	$\overline{A}.\overline{B}.\overline{C}.D$	$\overline{A}.\overline{B}.C.D$	$\overline{A}.\overline{B}.C.\overline{D}$
01	$\overline{A}.B.\overline{C}.\overline{D}$	$\overline{A}.B.\overline{C}.D$	$\overline{A}.B.C.D$	$\overline{A}.B.C.\overline{D}$
11	$A.B.\overline{C}.\overline{D}$	$A.B.\overline{C}.D$	$A.B.C.D$	$A.B.C.\overline{D}$
10	$A.\overline{B}.\overline{C}.\overline{D}$	$A.\overline{B}.\overline{C}.D$	$A.\overline{B}.C.D$	$A.\overline{B}.C.\overline{D}$

  - Four corners adjacent



Karnaugh Maps

- Four Variable K-Map example

$$F = \overline{A}.\overline{B}.\overline{C}.\overline{D} + \overline{A}.B.\overline{C}.D + \overline{A}.B.\overline{C}.\overline{D} + A.\overline{B}.\overline{C}.\overline{D} + \overline{A}.\overline{B}.C.\overline{D} + A.\overline{B}.C.\overline{D} + \overline{A}.\overline{B}.\overline{C}.D$$

AB \ CD	00	01	11	10
00				
01				
11				
10				

F =

Karnaugh Maps

- Four Variable K-Map solution

$$F = \overline{A}.\overline{B}.\overline{C}.\overline{D} + \overline{A}.B.\overline{C}.D + \overline{A}.B.\overline{C}.\overline{D} + A.\overline{B}.\overline{C}.\overline{D} + \overline{A}.\overline{B}.C.\overline{D} + A.\overline{B}.C.\overline{D} + \overline{A}.\overline{B}.\overline{C}.D$$

AB \ CD	00	01	11	10
00	1	1		1
01	1	1		
11				
10	1			1

$$F = \overline{B}.\overline{D} + \overline{A}.\overline{C}$$